The extended of 16th Hilbert probleblem for the discontinuous piecewise differential systems formed by quartic Hamiltonien centers and linear center separated by a straight line

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The study of the maximum number of limit cycles is one of the big problems in the qualitative theory of planar differential systems. In this paper we solve the 16th Hilbert problem for discontinuous piecewise differential system formed by linear center and Hamiltonian quartic centers and separated by a straight line x = 0. So we show the maximum number of limit cycles of this planar discontinuous piecewise differential system.

Any *planar* polynomial differential system takes the form $\dot{x} = P(x, y),\ \dot{y} = Q(x, y)$, where $P(x, y)$ and $Q(x, y)$ are polynomial functions, the *degree* of this system is the maximum degree of these polynomials.

The study of the existence and determination of the upper bound of the maximum number of *limit cycles* of planar polynomial differential systems is an attractive research topic that, until know, is still an unsolved problem in the qualitative theory of differential systems. As one of the 23 problems presented at the international congress of mathematicians in Paris in 1900, this problem is known as the second part of the sixteenth Hilbert problem. For more information, read, for example [3, 4].

This paper deals with piecewise discontinuous differential systems of the form

$$(\dot{x}, \dot{y}) = F(x, y) = \begin{cases} F^-(x, y) = \Big(F_1^-(x, y), F_2^-(x, y)\Big)^T & \mathbf{y} \in \Sigma^-, \\ F^+(x, y) = \Big(F_1^+(x, y), F_2^+(x, y)\Big)^T & \mathbf{y} \in \Sigma^+, \end{cases}$$

such that the separation line of the plane is $\Sigma = \{(x, y) : x = 0\}$ and

$$\Sigma^- = \{(x, y) : x \leq 0\}, \quad \Sigma^+ = \{(x, y) : x \geq 0\}.$$

In this paper we shall work with discontinuous piecewise differential systems in $\mathbb{R}^2$, and the definition of these differential systems on the separation line of their two pieces in $\mathbb{R}^2$ follow the rules of Filippov.

The second part of the famous sixteenth Hilbert problem consists in finding an upper bound for the maximum number of limit cycles that the polynomial differential systems in the plane of a given degree can have, see [2, 4]. In the last years many authors have been involved in solving the extension of this problem to some classes of discontinuous piecewise differential systems.

In the literature we find many papers interested in studying piecewise differential linear systems separated by either a straight line or an algebraic curve, such as a conic or a reducible or irreducible cubic curve, see for instance [1].

The main goal of this paper is to solve the extension of the second part of the sixteenth Hilbert problem to the class of discontinuous piecewise differential systems formed by linear center and an arbitrary Hamiltonian quartic centers separated by the straight line $x = 0$.

Theorem 1. *The maximum number of crossing limit cycles for the classes of discontinuous piecewise differential systems separated by $x = 0$, and formed by an arbitrary linear differential center in one region, and by one of the five classes of the Hamiltonian system of linear plus quartic homogeneous polynomials is at most two. There are examples with exactly two limit cycles for each class.*

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