Mild solutions for nonlocal neutral functional perturbed pseudo integrodifferential evolution equations with Finite state-dependent delay

Imane Abibssi[1], Selma Baghli-Bendimerad[2]
[1]University of Djillali Liabes, Sidi Bel Abbès, Algeria
[2]University of Djillali Liabes, Sidi Bel Abbès, Algeria
abibssi.imene@gmail.com, selmabaghli@gmail.com

In this work, we study the existence of mild solutions defined on the semi-infinite real interval $J := [0, +\infty)$, for a class of first order neutral functional evolution equations with finite state-dependent delay and nonlocal conditions in a real Banach space $(E, |\cdot|)$.

We consider the following nonlocal neutral functional perturbed pseudo integrodifferential evolution equation

$$\frac{d}{dt}[y(t) - Q(t, y_{\rho(t,y_t)})] = A(t)y(t) + f(t, y_{\rho(t,y_t)}) + \int_0^t \mathcal{I}(t,s) g\left(s, y_{\rho(s,y_s)}\right) ds, \quad \text{a.e.} \quad t \in J \quad (1)$$

$$y(t) + h_t(y) = \varphi(t), \quad \text{a.e.} \quad t \in H := [-r, 0] \quad r > 0, \quad (2)$$

where $f, g, Q : J \times C(H; E) \to E$, $\mathcal{I} : J \times J \to \mathbb{R}$, $\rho : J \times C(H; E) \to \mathbb{R}$
$h_t : C(H; E) \to E$ and $\varphi \in C(H; E)$ are given functions and $\{A(t)\}_{t\geq 0}$ is a family of linear closed (not necessarily bounded) operators from E into E that generates unique evolution system of operators $\{U(t,s)\}_{(t,s)\in J\times J}$ for $s \leq t$.

For any continuous function y defined on $[-r, +\infty)$ and any $t \in J$, we denote by y_t the element of $C(H, E)$ defined by

$$y_t(\theta) = y(t + \theta) \quad \text{for } \theta \in H.$$

Here $y_t(\cdot)$ represents the history of the state from time $t - r$ up to the present time t.

Our main purpose in this paper is to sufficient conditions for the existence of mild solutions on a semi infinite interval $J = [0; +\infty)$ for neutral perturbed pseudo integrodifferential evolution equations with state-dependent delay when the conditions are nonlocal (1)-(2) using the nonlinear alternative of Avramescu for a sum of compact operators and contractions maps in Fréchet spaces [1], combined with semigroup theory [3].

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