Stabilisation and existence of non linear Euler-Bernoulli beam

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In this paper, the free transverse vibration of a non linear Euler Bernoulli beam under a neutral type delay is considered. In order to suppress the beam transverse vibrations, a boundary control based on the Lyapunov method is designed. The novelty of this work is the ability to get a wide variety of energy decay rates under free vibration conditions.

Due to the requirement for high-precision control of numerous mechanical systems, such as marine risers for oil and gas transportation, spacecraft with flexible attachments, or flexible robot arms, the boundary control of flexible systems has been an important topic of study in recent years. The time delay is one of several elements that have a significant impact on the dynamic properties of systems. It became evident that its existence could not be fully neglected in many systems, and with the rapid growth of numerous engineering disciplines, including mechanical engineering, a more precise system analysis was necessary. We consider in this work the neutrally retarded nonlinear Euler-Bernoulli beam for $(x,t) \in (0,L) \times [0,\infty), L > 0$

$$\rho A\left[u_t + \int_0^t \kappa(t-s)u_t(x,s)ds\right]_t + EIu_{xxxx} - P_0 u_{xx} - \frac{1}{2}EA(u_x^3)_x = 0, \tag{1}$$

under the boundary

$$\begin{cases} u_{xx}(0,t) = u_{xx}(L,t) = u(0,t) = 0, \quad \forall\, t \geq 0, \\ EIu_{xxx}(L,t) = P_0 u_x(L,t) + \frac{1}{2}EAu_x^3(L,t) + \alpha u_t(L,t), \quad \forall\, t \geq 0, \quad \alpha > 0, \end{cases} \tag{2}$$

and initial conditions

$$u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \quad x \in (0,L), \tag{3}$$

where EI is the beam's flexural rigidity, ρA is the beam's mass per unit length, and $u(x,t)$ represents transverse displacement at time t with respect to the spatial coordinate x, EA the axial stiffness, P_0 the tension force. In this work we consider the transverse dynamics of a beam in bending vibration and we neglect the coupling between longitudinal and transversal displacements. Assuming that the change in length due to the axial force is small and negligible, we take only the elongation of the beam due to the curvature. We prove a general decay result for problem (1)–(3).

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