Variational analysis of an electro-viscoplastic contact problem

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In this work, we consider a dynamic process of frictional contact bilateral between a deformable body and an obstacle (see [2]). The material obeys an thermo-viscoplastic constitutive law . We derive a variational formulation of the problem which includes a variational second order evolution inequality. We establish the existence of a unique weak solution of the problem. The idea is to reduce the second order evolution nonlinear inequality of the system to first order evolution inequality (see [3,4]).

We consider a mathematical model of a contact problem in electro-viscoplastic (see [1])

$$\sigma(t) = \mathcal{A}(\varepsilon(\dot{u}(t))) + \mathcal{B}(\varepsilon(u(t)), \beta(t)) + \int_0^t \mathcal{G}\left(\sigma(s) - \mathcal{A}(\varepsilon(\dot{u}(s))), \varepsilon(u(s))\right) ds - \xi^* E(\varphi),$$
$$D = BE(\varphi) + \xi\varepsilon(u), \text{in } \Omega \times [0,T],$$

Where $u : \Omega \times [0,T] \to \mathbb{R}^d$is a displacement field, $\sigma : \Omega \times [0,T] \to \mathbb{S}^d$is a stress field $\varphi : \Omega \times [0,T] \to \mathbb{R}$is the an electric potentiel field, $D : \Omega \times [0,T] \to \mathbb{R}^d$is the an electric displacement field

The body is in contact with an obstacle. The contact is frictional and bilateral with a moving rigid foundation.

$$\begin{cases} \sigma_\nu = -\alpha\left|\dot{u}_\nu\right|, \ \ |\sigma_\tau| = -\mu\sigma_\nu, \\ \sigma_\tau = -\lambda\left(\dot{u}_\tau - v^*\right), \ \lambda \geq 0, \text{ on } \Gamma_3 \ \times [0,T], \end{cases}$$

We establish a variational formulation for the model and we prove the existence of a unique weak solution to the problem.

$$\begin{aligned} &(\ddot{u}(t), w - \dot{u}(t))_{V'\times V} + (\sigma(t), \epsilon(w - \dot{u}(t)))_{\mathcal{H}} + \\ &j(\dot{u}(t), w) - j(\dot{u}, \dot{u}(t)) + \phi(\dot{u}, w) - \\ &\phi(\dot{u}, \dot{u}(t)) \geq (f(t), w - \dot{u}(t)), \forall u, w \in V, \\ &(D(t), \nabla\varphi)_{\mathbb{L}^2(\Omega)^d} + (q(t), \varphi)_W = 0, \text{ forall } \varphi \in W \end{aligned}$$

The proof is based on a classical existence and uniqueness result on parabolic inequalities, differential equations and fixed point arguments.

Our main results on existence and uniqueness which state the unique weak solvability.

Theorem 1. *The Problem PV has a unique solution (u, σ, φ, D) which satisfies*

$$u \in C^1(0.T, H) \cap W^{1.2}(0.T, V) \cap W^{2.2}(0.T, V') \tag{1}$$
$$\sigma \in \mathbb{L}^2(0.T, \mathcal{H}_1), Div\sigma \in \mathbb{L}^2(0.T, V') \tag{2}$$
$$\varphi \in W^{1.2}(0.T, W) \tag{3}$$
$$D \in W^{1.2}(0.T, \mathcal{W}_1) \tag{4}$$

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