Evolution of correlations in a system of hard spheres

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This talk discusses with the mathematical problems of the description of the evolution of many hard spheres based on various ways of describing their state, namely by means of functions describing the propagation of correlations. One of the developed approaches allows one to describe the evolution of both a finite and an infinite average number of hard spheres using reduced distribution functions or reduced correlation functions, which are determined by the dynamics of correlations of a hard-sphere system [1]. We note the importance of the description of the processes of the creation and propagation of correlations [2], in particular, it is related to the problem of the description of the memory effects in many-particle systems with collision dynamics.

It was established that the notion of cumulants of the groups of operators underlies nonperturbative expansions of solutions for the fundamental evolution equations describing the state evolution of a hard-sphere system, namely, of the Liouville hierarchy for correlation functions, of the BBGKY hierarchy for reduced distribution functions and of the nonlinear BBGKY hierarchy for reduced correlation functions, as well as it underlies the kinetic description of infinitely many hard spheres. We emphasize that the structure of obtained expansions for correlation functions, in which the generating operators are the cumulants of the corresponding order of the groups of operators of hard spheres, induces the cumulant structure of series expansions for reduced distribution functions, reduced correlation functions and reduced correlation functionals. Thus, the dynamics of systems of infinitely many hard spheres is generated by the dynamics of correlations.

The origin of the collective behavior of a hard-sphere system on a microscopic scale was described by means of a one-particle correlation function that is determined by the non-Markovian Enskog kinetic equation. The advantages of such an approach to the derivation of kinetic equations from underlying collisional dynamics consists of an opportunity to construct the kinetic equations with initial correlations, which makes it possible to describe the propagation of initial correlations in the Boltzmann-Grad limit [3]. Another advantage of this approach is related to the rigorous derivation of the Boltzmann equation with higher-order corrections to the main term of the Boltzmann-Grad asymptotics of collisional dynamics.

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