Existence of solutions for a plate equation with a source term and non-local boundary condition

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In this work, we study the well posedness of a plate equation with a source term and non homogenuous boundary condition. The existence of solutions for the plate equation is proved by using the Faedo-Galerkin method.

Let Ω be an open bounded domain in $\mathbb{R}^n$ with a smooth boundary Γ and consider the following problem of a wave equation with non homogeneous boundary conditions

$$(\mathcal{P}) = \begin{cases} u'' - \Delta^2 u - u|u|^{\rho} = 0 \ \text{ in } \Omega \times (0,T), \\ \\ u(x,t) = f(x,t) \ \text{ on } \Gamma \times (0,T), \\ \\ u(x,0) = u^0, \ \ u'(x,0) = u^1 \ \text{ in } \Omega, \end{cases} \tag{1}$$

where Δ denotes the Laplacian operator with respect to the variable x, Δ^2 denotes the Bi-Laplacian operator and $u|u|^{\rho}$ represents the source term in the interior. We assume that

$$u^0 \in H_0^2(\Omega). \tag{2}$$

$$u^1 \in H_0^1(\Omega). \tag{3}$$

$$f \in L^2(\Gamma \times (0,T)). \tag{4}$$

Problem (1) is a particular case of a large class of partial differential equations called hyperbolic equations. For general results concerning this kind of equations, we refer the reader to Lions and Magenes (see [2,3]).

In order to prove the existence, we use Faedo-Galerkin method (see [1,4]). The method is based on the fact that the Hilbert space $H_0^1(\Omega)$ can be approximated by a sequence of finite dimensional sub spaces $\{V_m\}$ as $m \to \infty$.

The following Lemma will be used in the proof.

Lemma 1. *(Gronwall lemma) Let v be a function in $C([0,T])$, we suppose that there exists a positive constant k such that*

$$v(t) \leq k + \int_0^t v(s)ds, \quad \forall t \in [0,T].$$

Then

$$v(t) \leq ke^T, \quad \forall t \in [0,T].$$

The theorem of existence is as follows.

Theorem 1. *Assume that (2)–(4) are satisfied. Then, there exists a solution u of problem* (1) *such that*

$$(u,u') \in L^{\infty}(0,T;H_0^2(\Omega)) \times L^{\infty}(0,T;H_0^1(\Omega)).$$

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