on Stability and Stabilization of Perturbed Time Scale Systems with Gronwall Inequalities

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In this paper, we provide some sufficient conditions for the asymptotic stability of solutions of nonlinear dynamic systems with impulsive perturbations on time scale by using some inequality of Gronwall type. In this paper, we shall consider linear differential systems with impulsive perturbations modeled by

$$z^{\Delta} = A(t)\,z + F(t,z) + \psi(t,z)\,\Delta g, \tag{1}$$

where $z \in \mathbb{R}^n$, $t \in \mathbb{T}$ and $A : \mathbb{T} \to \mathbb{R}^{n*n}$ a regressive matrix-valued function , Δg denotes the distributional derivative of the function g. The functions $F, \psi : \mathbb{T} \times \mathbb{R}^n \to \mathbb{R}^n$ and $g : \mathbb{T} \cap [0,\infty) \to \mathbb{R}$ is a function of bounded variation, right-continuous on $\mathbb{T} \cap [0,\infty)$. Here Δg can be identified to a Stieltjes measure and will have the effect of sudden change of the state of the system at the points of discontinuity of g. Equations of the form (1) are called measure differential equations. Our interest here is to treat Eq. (1) as a perturbation of the linear differential system

$$z^{\Delta}(t) = A(t)\,z \tag{2}$$

where the perturbation $F(t,z) + \psi(t,z)\,\Delta g$, is impulsive

Let $z(t,t_0,z_0)$ and $y(t) = z(t,t_0,z_0)$ be the solutions of (2) and (1), respectively, through (t_0,z_0), existing in the right of $t_0 > 0$, The function $z(t,t_0,z_0)$ is a solution of (1) with $y(t_0) = z_0$, if and only if, it is a solution of

$$y(t) = R_A(t,t_0)\,z_0 + \int_{t_0}^{t} R_A(t,\sigma(s))\,F(s,y(s))\,\Delta(s) + \int_{t_0}^{t} R_A(t,\sigma(s))\,\psi(s,y(s))\,\Delta g(s)\,, t \in \mathbb{R}_+ \tag{3}$$

where $R_A(t,t_0)$ is the fundamental matrix of the equation (2) satisfying $R_A(t_0,t_0) = I$, where I denotes the unit matrix .

Definition 1. Let $r \geq 0$ and $B_r = \{z \in \mathbb{R}^n \diagup \|z\| \leq r\}$.

(i) The ball B_r is globally exponentially stable if there exist $\gamma > 0$ and $c > 0$, such that $\forall z_0 \in \mathbb{R}^n$

$$\|z(t)\| \leq c\,\|z_0\|\, e_{\ominus\gamma}(t,t_0) + r, \forall t \geq t_0$$

(ii) The system (1) is globally practically exponentially stable if there exists $r > 0$ such that B_r is globally exponentially stable.

Naturally, the size of the radius r depends on the upper bound of the perturbation term.

We shall assume throughout all the paper that the nominal system

$$z^{\Delta} = A(t)\,z,\ z(t_0) = z_0$$

is uniformly asymptotically stable (U.A.S), if and only if there exists a positive constant a, with $-a \in \mathcal{R}^{+}$and there is $c \geq 1$ independent on any initial point t_0, such that

$$\|R_A(t,t_0)\| \leq c e_{\ominus a}(t,t_0) \text{ for all } t \in \mathbb{T}^{+}_{t_0}. \tag{4}$$

Theorem 1. *Assume that:*
(i) *the nominal system* (2) *is U.A.S,*
(ii) F *satisfies a Lipschitz condition of the type*

$$\|F(t,z) - F(t,z)\| \leq L(t)\|z - y\|$$

where $L(t) \in L^p(\mathbb{T}\cap\mathbb{R}_+, \mathbb{T}\cap\mathbb{R}_+)$ *with* $p \in \mathbb{T}\cap[1,\infty)$,
(iii) $\|\psi(t,z)\| \leq \gamma(t)$ *for all* $t \geq 0$, *where* $\gamma(t)$ *is a continuous function and*

$$G(t) = \int_t^{t+1} \gamma(s)\,\Delta v(s) \to 0,\ t \to \infty \tag{5}$$

Here $v(t)$ *is the total variation of the function* g *over the interval* $\mathbb{T}\cap[0,t]$.

Then, if $\psi(t,0) = 0$, *the null solution of* (1) *is globally asymptotically stable under the condition* $\|L\|_p \geq p\left(1 - \frac{a}{c}\right)$ *if* $p \in \mathbb{T}\cap(1,\infty)$.

Theorem 2. *Assume that all the conditions of Theorem 1 are satisfied except that condition* (ii) *is replaced by*

$$\|F(t,z)\| \leq \xi(t)\|z\| + \varsigma(t)$$

where ξ*and* $\varsigma \in Lp(\mathbb{R}_+, \mathbb{R}_+)$ *with* $p \in [1,\infty]$. *Then the system* (1) *is globally practically exponentially stable under the condition* $\|\xi\| > p\left(1 - \frac{a}{c}\right)$ *in the case* $p \in (1,\infty)$. *The result is still valid in case* $p = \infty$ *under the additional condition* $\|\xi\|_\infty < \frac{a}{c}$

1. Abraehim. A. K, Jaber. A. K, and Al-Salih. R, Flow optimization in dynamic networks on time scales, J. Phys. Conf. Ser., vol. 1804, no. 1, 7 pages, 2021, https://doi.org/10.1088/1742-6596/1804/1/012025.
2. Akın-Bohner.E. , Raffoul, Y. N, and Tisdell. C.C, Exponential stability in functional dynamic equations on time scales, Commun. Math. Anal., vol. 9, no 1, pp. 93–108, 2010. [15] A. C. Peterson and Y. N. Raffoul, Exponential stability of dynamic equations on time scales, Adv. Difference Equ., vol. 2005, no. 2, 2005, Article ID 858671, https://doi.org/10.1155/ADE.2005.133.
3. Atıcı. F. M, Biles. D. C, and Lebedinsky. A, An application of time scales to economics, Math. Comput. Model., vol. 43, no. (7–8), pp. 718–726, 2006, https://doi.org/10.1016/j.mcm.2005.08.014.