Ulam-Hyers-Rassias stability of boundary value discrete fractional hybrid equation

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In the problem, we will study the stability of solutions to the hybrid discrete Riemann-Liouville fractional problem [2]

$$\begin{cases} {}^{Rl}\Delta_{\zeta}^{\eth}\left[\dfrac{\wp^*(\zeta)}{\mathfrak{G}(\zeta,\wp^*(\zeta))}\right] = \Upsilon(\zeta+\eth-1,\wp^*(\zeta+\eth-1)), \quad 2<\eth\leq 3 \\ \Delta\left[\dfrac{\wp^*(\zeta)}{\mathfrak{G}(\zeta,\wp^*(\zeta))}\right]_{\zeta=\eth-3} = \mathfrak{A}_1, \ \left[\dfrac{\wp^*(\zeta)}{\mathfrak{G}(\zeta,\wp^*(\zeta))}\right]_{\zeta=\eth+\beth} = \lambda\Delta^{-\sigma}\left[\dfrac{\wp^*(\zeta)}{\mathfrak{G}(\zeta,\wp^*(\zeta))}\right]_{\zeta=\hbar+\sigma} \\ \Delta^2\left[\dfrac{\wp^*(\zeta)}{\mathfrak{G}(\zeta,\wp^*(\zeta))}\right]_{\zeta=\eth-3} = \mathfrak{A}_2, \end{cases} \tag{1}$$

for $0<\sigma\leq 1$, $\zeta\in[0,\beth]_{\mathbb{N}_0}=[0,1,...,\beth]$ and $\hbar\in[\eth-1,\beth+\eth-1]_{\mathbb{N}_{\eth-1}}$, where ${}^{Rl}\Delta_{\zeta}^{\eth}$ is the Riemann-Liouville fractional difference operator (RLFDO).

Environments:

Theorem 1. [2] *Let* $2<\eth\leq 3$ *and* $\Upsilon:[\eth-3,\eth+\beth]_{\mathbb{N}_{\eth-3}}\longrightarrow\mathbb{R}$ *be given. Then a function* $\wp^*$ *is a solution to the discrete HFBVP(1) if and only if* $\wp^*(\zeta)$*, for* $\zeta\in[\eth-3,\eth+\beth]_{\mathbb{N}_{\eth-3}}$ *is a solution to the following integral equation:*

$$\begin{aligned}\wp^*(\zeta) = \mathfrak{G}(\zeta,\wp^*(\zeta))\Bigg[&\frac{1}{\Gamma(\eth)}\sum_{l=0}^{\zeta-\eth}(\zeta-\iota(l))^{(\eth-1)}\Upsilon(l+\eth-1)\\ &+\left\{\frac{[\mathfrak{A}_2-\mathfrak{A}_1(\eth-3)]}{\Gamma(\eth)}+\frac{\frac{\mathfrak{F}_\Upsilon+\mathfrak{D}_\eth}{\mathfrak{H}_\eth}(\eth-3)}{2(\eth-1)}\right\}\zeta^{(\eth-1)}\\ &+\frac{\left[\mathfrak{A}_1-\frac{1}{\mathfrak{H}_\eth}[\mathfrak{F}_\Upsilon+\mathfrak{D}_\eth](\eth-3)\Gamma(\eth-2)\right]}{\Gamma(\eth-1)}\zeta^{(\eth-2)}\\ &+\frac{\mathfrak{F}_\Upsilon+\mathfrak{D}_\eth}{\mathfrak{H}_\eth}\zeta^{(\eth-3)}\Bigg],\end{aligned}$$

where

$$\begin{aligned}\mathfrak{H}_\eth =& \frac{\eth-3}{(\eth-2)}(\eth+\beth)^{(\eth-2)}-\frac{\eth-3}{2(\eth-1)}(\eth+\beth)^{(\eth-1)}-(\eth+\beth)^{(\eth-3)}\\ &+\frac{\lambda(\eth-3)}{\Gamma(\sigma)2(\eth-1)}\sum_{l=\eth-3}^{\hbar}(\hbar+\sigma-\iota(l))^{(\sigma-1)}l^{(\eth-1)}-\frac{\lambda(\eth-3)}{\Gamma(\sigma)(\eth-2)}\sum_{l=\eth-3}^{\hbar}(\hbar+\sigma-\iota(l))^{(\sigma-1)}l^{(\eth-2)}\\ &+\frac{\lambda}{\Gamma(\sigma)}\sum_{l=\eth-3}^{\hbar}(\hbar+\sigma-\iota(l))^{(\sigma-1)}l^{(\eth-3)}\end{aligned}$$

$$\mathfrak{D}_{\eth} = \frac{\mathfrak{A}_1(\eth+\beth)^{(\eth-2)}}{\Gamma(\eth-1)} - \frac{[\mathfrak{A}_2 - \mathfrak{A}_1(\eth-3)]\lambda}{\Gamma(\eth)\Gamma(\sigma)} \sum_{l=\eth-3}^{\hbar} (\hbar+\sigma-\iota(l))^{(\sigma-1)} l^{(\eth-1)}$$

$$- \frac{\mathfrak{A}_1\lambda}{\Gamma(\eth-1)\Gamma(\sigma)} \sum_{l=\eth-3}^{\hbar} (\hbar+\sigma-\iota(l))^{(\sigma-1)} l^{(\eth-2)} + \frac{[\mathfrak{A}_2 - \mathfrak{A}_1(\eth-3)]}{\Gamma(\eth)} (\eth+\beth)^{(\eth-1)}$$

$$\mathfrak{F}\Upsilon = -\frac{\lambda}{\Gamma(\eth)\Gamma(\sigma)} \sum_{l=\eth}^{\hbar} \sum_{\zeta=0}^{l-\eth} (\hbar+\sigma-\iota(l))^{(\sigma-1)} (l-\iota(\zeta))^{(\eth-1)} \Upsilon(\zeta+\eth-1)$$

$$+ \frac{1}{\Gamma(\eth)} \sum_{l=0}^{\beth} (\eth+\beth-\iota(l))^{(\eth-1)} \Upsilon(l+\eth-1)$$

Definition 1. [1] The discrete FBVP (1) is said to be Hyers-Ulam-stable If for every function $\varpi \in C(\mathbb{N}_{\eth-3,\eth+\beth}, \mathbb{R})$ of

$$\left| {}^{Rl}\Delta_{\zeta}^{\eth} \left[\frac{\varpi(\zeta)}{\mathfrak{G}(\zeta, \varpi(\zeta))} \right] - \Upsilon(\zeta+\eth-1, \varpi(\zeta+\eth-1)) \right| \le \epsilon, \quad \zeta \in [0, \beth]_{\mathbb{N}_0}$$

and $\epsilon > 0$, there exists a solution $\wp^* \in C(\mathbb{N}_{\eth-3,\eth+\beth}, \mathbb{R})$ of (1) and $\delta > 0$ such that

$$\left| \varpi(\zeta) - \wp^*(\zeta+\eth-1) \right| \le \delta\epsilon, \quad \zeta \in [\eth-3, \eth+\beth]_{\mathbb{N}_{\eth-3}}$$

Definition 2. [3] The discrete FBVP (1) is said to be Hyers-Ulam-Rassias stable If for every function $\varpi \in C(\mathbb{N}_{\eth-3,\eth+\beth}, \mathbb{R})$ of

$$\left| {}^{Rl}\Delta_{\zeta}^{\eth} \left[\frac{\varpi(\zeta)}{\mathfrak{G}(\zeta, \varpi(\zeta))} \right] - \Upsilon(\zeta+\eth-1, \varpi(\zeta+\eth-1)) \right| \le \epsilon\varphi(\zeta+\eth-1), \quad \zeta \in [0, \beth]_{\mathbb{N}_0}$$

and $\epsilon > 0$, there exists a solution $\wp^* \in C(\mathbb{N}_{\eth-3,\eth+\beth}, \mathbb{R})$ of (1) and $\delta_2 > 0$ such that

$$\left| \varpi(\zeta) - \wp^*(\zeta+\eth-1) \right| \le \delta_2\epsilon\varphi(\zeta+\eth-1), \quad \zeta \in [\eth-3, \eth+\beth]_{\mathbb{N}_{\eth-2}}$$

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