International Conference of Young Mathematicians
The Institute of Mathematics of the National Academy of Sciences of Ukraine
June 1–3, 2023, Kyiv, Ukraine

Numerical solutions of nonlinear fractional differntail equation using a modified ADM

Chinoune Hanane
chinonhanan@gmail.com

In this work, we discusses a revised form of a modified the Adomian decomposition method [1, 2] technique that is applicable to solving a particular class of mathematical equations, when given initial conditions. Then we have presented several numerical examples to illustrate the effectiveness of this approach.

We present nonlinear equation with given initial conditions as follows:

$$\begin{cases} {}^{c}\mathcal{D}^{\ell}_{0^+}u(x) & +\sum_{i=0}^{\mathbf{m}}\lambda_i u^{(i)}(x)+\mu f\big(u(x),u'(x),...,u^{(\mathbf{m})}(x)\big)=\phi(x), \quad x>0, \\ & u(0)=\mathbf{d}_0,\ \ u'(0)=\mathbf{d}_1,...,\ \ u^{(\mathbf{m})}(0)=\mathbf{d}_{\mathbf{m}}, \end{cases} \tag{1}$$

where $\mathbf{m}\geq 1$, $\mathbf{m}<\ell\leq \mathbf{m}+1$, $a,b,\mu,\lambda_i,\mathbf{d}_i$ $(i=0,1,...,\mathbf{m})$ are known constant real numbers, $f:\mathbb{R}^{n+1}\to\mathbb{R}$ is a continuous function and ${}^{c}\mathcal{D}^{\ell}_{0^+}$ stands the Caputo fractional derivative of order ℓ .

Definition 1 ([1])**.** The fractional integration of order $\varrho>0$ in the Riemann-Liouville for a continous function $\chi:\mathbb{R}^+\to\mathbb{R}$ is expressed as

$$\mathcal{I}^{\varrho}_{0^+}\chi(t)=\frac{1}{\Gamma(\varrho)}\int_0^t (t-\tau)^{\varrho-1}\chi(\tau)d\tau, \tag{2}$$

Definition 2 ([1])**.** The Caputo derivation of order $\varrho>0$ for a continous function $\chi:\mathbb{R}^+\to\mathbb{R}$ is given by

$${}^{c}\mathcal{D}^{\varrho}_{0^+}\chi(t)=\frac{1}{\Gamma(n-\varrho)}\int_0^t (t-\tau)^{n-\varrho-1}\chi^{(n)}(\tau)d\tau, \tag{3}$$

such that the integral (3) has a finite value, with $n-1<\varrho\leq n$.

Presentation of the modified version of ADM.
We consider the following operator equation [3]

$$\mathcal{L}u+\mathcal{R}u+\mathcal{N}u=\phi, \tag{4}$$

where

- $\mathcal{L}$ denotes an invertible operator,
- $\mathcal{R}$ represents the linear operator,
- $\mathcal{N}$ stands the nonlinear terms,
- ϕ is a known supposed function such that $\mathcal{L}^{-1}(\phi)$ exists.

Taking the inverse operator $\mathcal{L}^{-1}$ on two members of (4), we get

$$u = \delta + \mathcal{L}^{-1}(\phi) - \mathcal{L}^{-1}(\mathcal{R}u) - \mathcal{L}^{-1}(\mathcal{N}u),$$

where δ can be calculated from the introduced initial values. In this method, we decompose $u(x)$ into convergent series, and $\mathcal{N}u(x)$ into the series of the Adomian polynomials

$$u(x) = \sum_{n=0}^{\infty} u_n(x),$$

$$\mathcal{N}u(x) = \sum_{n=0}^{\infty} \mathcal{A}_n,$$

such that $\mathcal{A}_n = \mathcal{A}_n(u_0(x), u_1(x), ..., u_n(x))$ stands the Adomian polynomials expressed

$$\mathcal{A}_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\mathcal{N}\left(\sum_{k=0}^{\infty} u_k\lambda^k\right)\Bigg|_{\lambda=0}, \quad n \geq 0.$$

Finally, the Adomian recursion scheme is given by

$$\begin{cases} u_0(x) = \delta(x) + \mathcal{L}^{-1}\phi(x), \\ u_{n+1}(x) = -\mathcal{L}^{-1}\big(\mathcal{R}u_n(x) + \mathcal{A}_n\big). \end{cases}$$

1. Kilbas A.A., Srivastava H.M., Trujillo J.J. Theory and Applications of Fractional Differential Equations. — Amsterdam: Elsevier, 2006, 540 p.
2. Jafari H., Daftardar-Gejji V. Positive solutions of nonlinear fractional boundary value problems using Adomian decomposition method. Appl. Math. Comput., 2006, 180, 700–706.
3. Daftardar-Gejji V., Jafari H. Adomian decomposition: a tool for solving a system of fractional differential equations. J. Math. Anal. Appl., 2005, 301, 508–518.