Numerical stability result for damped Shear beam model

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In this paper, we consider a beam model known as Shear beam model (no rotary inertia). we prove that the corresponding dissipative Shear beam model has an energy exponential decay regardless any relationship between coefficients of the system. Numerically, we construct a numerical scheme based on the P1-fnite element method for space discretization and implicit Euler scheme for time discretization. Finally, some numerical simulations are presented.

Motivated by [1] and the simplified beam model presented in paper [2], the aim of this paper is to investigate the effect of damped term on the behavior of the solution to the following system:

$$\begin{cases} \rho_1\varphi_{tt}(x,t) - K(\varphi_x+\psi)_x(x,t) + \mu\varphi_t(x,t) = 0, \\ -b\psi_{xx}(x,t) + K(\varphi_x+\psi)(x,t) = 0, \end{cases} \tag{1}$$

where $t \in (0,\infty)$ denotes the time variable and $x \in (0,1)$ is the space variable, the functions φ and ψ are respectively, the transverse displacement and the rotation of the neutral axis. ρ_1, b, μ and K are positive constants.

We consider the following initial conditions

$$\varphi(x,0) = \varphi_0(x), \qquad \varphi_t(x,0) = \varphi_1(x), \qquad \psi(x,0) = \psi_0(x) \tag{2}$$

and the boundary conditions

$$\varphi(0,t) = \varphi(1,t) = \psi(0,t) = \psi(1,t) = 0. \tag{3}$$

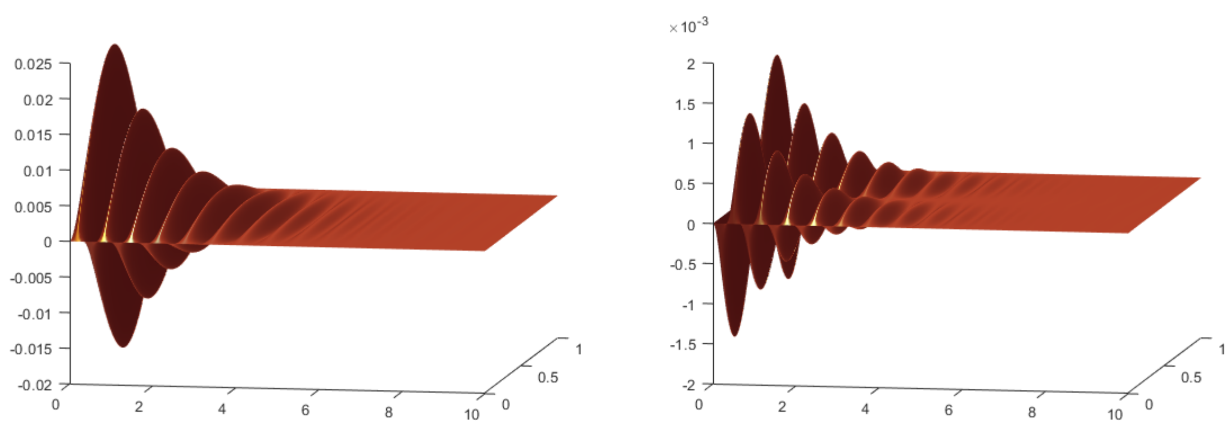

Figure 1: The evolution in time and space of φ and ψ.

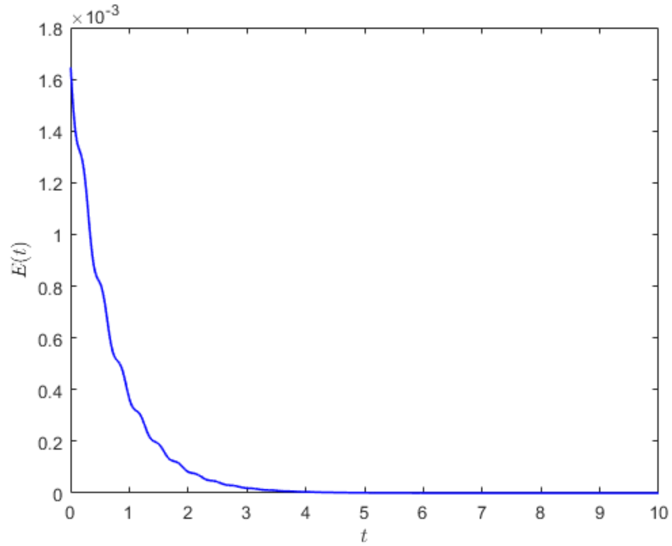

Figure 2: The evolution in time of E.

Definition 1. We define here the energy associated with the system (1)-(3) by

$$E(t)=\frac{1}{2}\int_0^1\left[\rho_1\varphi_t^2+K(\varphi_x+\psi)^2+b\psi_x^2\right]dx. \tag{4}$$

Theorem 1. *The energy (4) decays exponentially as time t goes to infinity. That is, there exist two positive constants C and γ independent of t such that*

$$E\left(t\right)\leq Ce^{-\gamma t},\qquad \forall\ t\geq 0.$$

As in [3, 4], we propose a finite element discretization of this problem. We present an iterative algorithm for solving the discrete problem. Finally, we obtain the following numerical simulations using MATLAB software.

1. Fernández Sare H. D., Racke R. On the stability of damped Timoshenko systems – cattaneo versus Fourier's law. Arch. Rational Mech. Anal., 2009, 194, No. 1, 221–251.
2. Han S. M., Benaroya H., Timothy Wei. Dynamics of transversely vibration beams using four engineering theories. J. Sound Vib, 1999, 225, No. 5, 935–988. DOI: 10.1006/jsvi.1999.2257.
3. Bernardi C., Copetti M. I. M. Discretization of a nonlinear dynamic thermoviscoelastic Timoshenko beam model. Z. Angew. Math. Mech., 2017, 97, 532–549.
4. El Arwadi T., Copetti M. I. M., Youssef W. On the theoretical and numerical stability of the thermoviscoelastic Bresse system. Z. Angew. Math. Mech., 2019, 99, No. 10, 1–20.