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Nonlocally controllability of mild solutions for neutral evolution problems with finite state-dependent delay

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In this work, we study the controllability of mild solutions defined on the semi-infinite real interval $J := [0, +\infty)$, for a class of first order neutral functional evolution equations with finite state-dependent delay and nonlocal conditions in a real Banach space $(E, |\cdot|)$.

We consider the following nonlocal neutral functional differential evolution equation

$$\frac{d}{dt}[y(t) - g(t, y_{\rho(t,y_t)})] = A(t)y(t) + Cu(t) + f(t, y_{\rho(t,y_t)}), \qquad a.e.\ t \in J, \tag{1}$$

$$y(t) + h_t(y) = \varphi(t), \qquad t \in H = [-r, 0], \tag{2}$$

where $r > 0$; $f, g : J \times C(H, E) \to E$, $\rho : J \times C(H, E) \to \mathbb{R}$, $h_t : C(H, E) \to E$ and $\varphi \in C(H, E)$ are given functions; the control function $u(\cdot)$ is given in $L^2(J, E)$ is the Banach space of admissible control function; C is a bounded linear operator from E into E and $\{A(t)\}_{t \in J}$ is a family of linear closed (not necessarily bounded) operators from E into E which generates an evolution system of operators $\{U(t, s)\}_{(t,s) \in J \times J}$ for $s \le t$.

For any continuous function y defined on $[-r, +\infty)$ and any $t \in J$, we denote by y_t the element of $C(H, E)$ defined by

$$y_t(\theta) = y(t + \theta) \quad \text{for } \theta \in H.$$

Here $y_t(\cdot)$ represents the history of the state from time $t - r$ up to the present time t.

Our purpose in this work is to give an extension for the previous controllability results obtained by Baghli *et al.* specially in [3] with nonlocal neutral problems (1)- (2). We provide sufficient conditions for the existence of mild controllable solutions using the nonlinear alternative of Avramescu [2] for sum of compact operators and contractions maps in Fréchet spaces, combined with semigroup theory [4].

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