On the positive fractional 2D continuous-time Lyapunov systems

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The aim of this talk is to introduce a new class of positive fractional 2D continuous-time linear system described by Lyapunov differential equation and based on the Roesser model. The solution of this system is derived. The positivity conditions are extended to the considered model. Necessary and sufficient conditions for the asymptotic stability of the positive 2D continuous-time Lyapunov systems are established. Finally, numerical examples are presented.

Problem 1. Motivated by the work of [2] and [3], which respectively study the problem of positive Linear Lyapunov systems in the 1D and 2D discrete cases, we investigate the stability of a positive fractional 2D continuous-time Lyapunov linear system. This system is described by the Roesser model [1], which is defined as:

$$\begin{aligned}
\begin{bmatrix} D^{\alpha}_{t_1} x^h(t_1,t_2) \\ D^{\beta}_{t_2} x^v(t_1,t_2) \end{bmatrix} =& \begin{bmatrix} A^0_{11} & A^0_{12} \\ A^0_{21} & A^0_{22} \end{bmatrix} \begin{bmatrix} x^h(t_1,t_2) \\ x^v(t_1,t_2) \end{bmatrix} + \begin{bmatrix} x^h(t_1,t_2) \\ x^v(t_1,t_2) \end{bmatrix} \begin{bmatrix} A^1_{11} & A^1_{12} \\ A^1_{21} & A^1_{22} \end{bmatrix} \\
&+ \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u(t_1,t_2) \\
y(t_1,t_2) =& \begin{bmatrix} C_1 & C_2 \end{bmatrix} \begin{bmatrix} x^h(t_1,t_2) \\ x^v(t_1,t_2) \end{bmatrix} + Du(t_1,t_2) \\
t_1,t_2 \in& \, \mathbb{R}_+
\end{aligned} \tag{1}$$

where $D^{\alpha}_{t_1}, D^{\beta}_{t_2}$ the Caputo fractional derivative operators, $0 < \alpha, \beta < 1$, $A^0_{11}, A^1_{11} \in \mathbb{R}^{n_1\times n_1}, A^0_{22}, A^1_{22} \in \mathbb{R}^{n_2\times n_2}, B_1 \in \mathbb{R}^{n_1\times m}, B_2 \in \mathbb{R}^{n_2\times m}, C_1 \in \mathbb{R}^{p\times n_1}, C_2 \in \mathbb{R}^{p\times n_2}, D \in \mathbb{R}^{p\times m}$. The matrices $x^h(t_1,t_2) \in \mathbb{R}^{n_1\times n}, x^v(t_1,t_2) \in \mathbb{R}^{n_2\times n}$ are, respectively, the horizontal and vertical matrices states at the point (t_1,t_2), $u(t_1,t_2) \in \mathbb{R}^{m\times n}$ is input matrix, $y(t_1,t_2) \in \mathbb{R}^{m\times n}$ is output matrixn and $n_1 + n_2 = n$. The boundary conditions of the system (1) are given by

$$\left\{ x^h(0,t_2), x^v(t_1,0) \text{ where } t_1 \in \mathbb{R}_+, t_2 \in \mathbb{R}_+ \right\}. \tag{2}$$

Based on the results of the positive fractional Roesser model presented in [7], we present the following resultat.

Theorem 1. *The fractional 2D Lyapunov system* (1) *is positive if and only if,* A^0 *and* A^1 *are the Metzler matrices, and* $B \in R^{n\times m}_+$, $C \in R^{p\times n}_+$, $D \in R^{p\times m}_+$.

Theorem 2. *The fractional 2D Lyapunov system* (1) *is symptotically stable if and only if any of the following equivalent conditions are satisfied:*

a) $\mathrm{Re}\,(\lambda_i + \mu_j) < 0$, *for* $i, j = 1, 2, \ldots, n$ *where* $\lambda_1, \lambda_2, \ldots \lambda_n$ *are the eigenvalues of the matrix* A^0 *and* $\mu_1, \mu_2, \ldots \mu_n$ *the eigenvalues of the matrix* A^1.

b) There is some strictly positive matrix $\theta \in \mathbb{R}^{n\times n}$ *such that*

$$A^0\theta + \theta A^1 < 0. \tag{3}$$

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