On the implicit stationary differential equations in Hilbert spaces

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The main purpose of this work is to extend te famous Lyapunov theorem of the linear explicit differential equations (continuous or discrete), for studying the spectrum of some implicit stationary differential equations by using the properties of the spectral theory for the corresponding operator pencil and an appropriate tronsformation. The results obtained can be applied to study the stabilizations for certain implicit controlled systems.

In the present paper, we consider the following implicit stationary differential equation of the form:

$$Ax'(t) = Bx(t), \quad t \geq t_0, \tag{1}$$

with initial condition:

$$x(t_0) = x_0,$$

where A and B are linear bounded operators acting in the same Hilbert space $\mathcal{H}$ also, the operator A is not necessarly invertible. We suppose that for each complex number λ, we have:

$$x(t) = e^{\lambda(t-t_0)}x_0,$$

which is a solution for (1) satisfies $(\lambda A - B)x_0 = 0$. At first, we denote by L the pencil of bounded operators A and B such that $L(\lambda) = \lambda A - B$ then, we recall that our main objective is to generalize the Lyapunov theorem for some equations such (1), and we use the obtained results to study the stabilization of a controlled implicit system described by the general form:

$$Ax'(t) - Bx(t) = Cu(t), \quad u \in U \subset \mathcal{H}, \quad t_0 \leq t \leq T. \tag{2}$$

Here u is control and T is a time, the operator C is also bounded on $\mathcal{H}$. In the sequel, we need the following definitions.

Definition 1. The complex number $\lambda \in \mathbb{C}$ is said to be a regular point of the pencil $\lambda A - B$, if the operator $(\lambda A - B)^{-1}$ exists and it is bounded. The set of all regular points is denoted by $\rho(A, B)$, and its complement $\sigma(A, B) = \mathbb{C} \backslash \rho(A, B)$ is called the spectrum of the pencil $\lambda A - B$. The set of all eigen-values of the pencil $\lambda A - B$ is denoted by:

$$\sigma_p(A, B) = \{\lambda \in \mathbb{C} \backslash \exists v \neq 0, (\lambda A - B)v = 0\}. \tag{3}$$

Definition 2. The implicit controlled system 2 is said to be exponentially stabilizable by means of a direct feedback $u(t) = Kx(t)$, if the given system:

$$Ax'(t) - (B + CK)x(t) = 0, \quad t_0 \leq t \leq T \tag{4}$$

is exponentially stable (K suppose to be a linear bounded operator in the Hilbert space $\mathcal{H}$).

Theorem 1. *Suppose taht the spectrum $\sigma(A,B)$ of the pencil of bounded operators A and B is in the left half-plane. Then, for any non-negative uniform operator $G \gg 0$, there exists an operator $W \gg 0$ such that:*

$$B^*WA + A^*WB = G.$$

Theorem 2. *In finite dimentional Hilbert spaces, the following assertions are equivalent:*

- *The sytem* (1) *is exponentially stabilizable,*
- $\sigma(A, B+CD) = \sigma_p(A, B+CD) \subset \{\lambda \in \mathbb{C}, \quad Rel(\lambda) < 0\}$,
- *There exists a non-negative de*

  nite matrix $W \gg 0$ such that:

$$(B+CD)^*WA + A^*W(B+CD) = G \gg 0.$$

Remark 1. We note that the similar results for discrete implicit systems are obtained in [7].

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