INVARIANT REGIONS AND ASYMPTOTIC BEHAVIOR FOR REACTION-DIFFUSION SYSTEMS WITH A TRIDIAGONAL SYMMETRIC TOEPLITZ MATRIX OF DIFFUSION COEFFICIENTS

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The aim of this article is to construct invariant regions of a generalized m -component reaction-diffusion system with tridiagonal symmetric Toeplitz diffusion matrix and homogeneous boundary conditions and polynomial growth for the nonlinear reaction terms. Using the eigenvalues and eigenvectors of the diffusion matrix and the parabolicity conditions. So we prove the asymptotic behavior of solutions in $C\left(\overline{\Omega}\right)\times C\left(\overline{\Omega}\right)\times ...\times C\left(\overline{\Omega}\right)$ and apply Lyapunov type stability techniques. A key ingredient in this analysis is a result which establishes that the orbits of the dynamical system are precompact in $C\left(\overline{\Omega}\right)\times C\left(\overline{\Omega}\right)\times ...\times C\left(\overline{\Omega}\right)$. As a consequence of Arzela-Ascoli theorem, this will be satisfied if the orbits are, for example, uniformly bounded in $C^1\left(\overline{\Omega}\right)\times C^1\left(\overline{\Omega}\right)\times ...\times C^1\left(\overline{\Omega}\right)$ for $t>0$. We consider the following m-equations of reaction-diffusion system, with $m\geq 2$:

$$\begin{cases} \frac{\partial U}{\partial t}-A_m\Delta U=F(U) & \text{in } \Omega\times(0,+\infty), \\ \partial_\eta U=0 & \text{on } \partial\Omega\times(0,+\infty), \\ U(0,x)=U_0(x)=(u_1^0,...,u_m^0)^T & \text{on } \Omega, \end{cases}$$

where Ω is an open bounded domain of class C^1 in $\mathbb{R}^n$, the vectors U and F and the matrix A_m are defined as:

$$U=(u_1,...,u_m)^T,\ F=(f_1,...,f_m)^T,$$

$$A_m=\begin{pmatrix} a_1 & b_1 & 0 & \cdots & 0 \\ c_1 & a_2 & b_2 & \ddots & \vdots \\ 0 & c_2 & a_3 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & b_{m-1} \\ 0 & \cdots & 0 & c_{m-1} & a_m \end{pmatrix}$$

the constants $(a_i)_{i=1}^m$, $(b_i)_{i=1}^{m-1}$ et $(c_i)_{i=1}^{m-1}$ are supposed to be strictly positive and satisfy the condition

$$\cos^2(\frac{\pi}{m+1})<\frac{a_ia_{i+1}}{(b_i+c_i)^2}$$

which reflects the parabolicity of the system and implies at the same time that the diffusion matrix A_m is positive defnite.

Theorem 1. *(See [3]) Let $T > 0$. A function $u : [0,T] \to X$ is a weak solution of*

$$\begin{cases} u_t(t) = Lu(t) + f(u(t)) \\ u(0) = u_0, \end{cases} \tag{1}$$

on $[0,T]$ if and only if $f(u(t)) \in L^1(0,T,X)$ and u satisfies the variation of constants formula

$$u(t) = S(t)\,u_0 + \int_0^t S(t-s)\,f(u(s))\,ds, \quad \text{for all } s \in [0,T].$$

Theorem 2. *(See [1]) Let $f : X \to X$ be locally Lipschitz continuous. Then for $u_0 \in X$, (1) has a unique weak solution defined in a maximal interval of existence $[0, T_{\max})$, $T_{\max} > 0$, $u \in C([0,T_{\max}), X)$. Moreover, if $T_{\max} < \infty$, then*

$$\lim_{t \to T_{\max}} \|u(t)\| = +\infty.$$

Now, let us recall the following definition.

Definition 1. (See [2]) Let $\{G(t)\}_{t\geq 0}$ be a nonlinear semigroup on a compact metric space X. If $(u_1^0, u_2^0, \ldots, u_m^0) \in X$, $O(u_1^0, u_2^0, \ldots, u_m^0) = \{G(t)(u_1^0, u_2^0, \ldots, u_m^0)\}_{t\geq 0}$ is the orbit through $(u_1^0, u_2^0, \ldots, u_m^0)$.

Then the w-limite set for $(u_1^0, u_2^0, \ldots, u_m^0)$ is defined by

$$w\left(u_1^0, u_2^0, \ldots, u_m^0\right) = \{(u_1, u_2, \ldots, u_m) \in X : \exists t_n \to \infty :$$

$$G(t_n)\left(u_1^0, u_2^0, \ldots, u_m^0\right) \to (u_1, u_2, \ldots, u_m)\}.$$

So in this article, we give some the main results.

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2. Mebarki M., A.Moumeni, Global solution of system reaction diffusion with full matrix. Global journal of mathematical analysis, 2015, 4–25.
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