Asymptotic behavior of a Viscoelastic Wave Equation with Time-Varying Delay in Fractional Internal Feedbacks.

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This paper explores the fascinating world of the viscoelastic wave equation with a time-varying delay term in internal fractional feedbacks. We present the problem as an equation with initial and boundary conditions of Dirichlet type, and establish the global existence of solutions by utilizing the energy method in conjunction with the Faedo-Galerkin procedure, provided certain conditions are met. We also demonstrate how suitable Lyapunov functionals can yield general decay results of the energy. This research is an essential contribution to our understanding of the viscoelastic wave equation, which has applications in various fields.

The viscoelastic wave equation is a classical model that has been extensively studied in the literature due to its significance in numerous areas of science and engineering, including materials science, acoustics, and geophysics, among others. In this paper, we consider a viscoelastic wave equation with a time-varying delay term in internal fractional feedbacks, which is described by the following equation:

$$\begin{cases} u_{tt}(x,t) - \Delta u(x,t) + \displaystyle\int_0^t g(t-s)\Delta u(x,s)ds + \\ \qquad \mu_1 u_t(x,t) + \mu_2 \partial_t^{\alpha,\beta} u(x,t-\tau) = 0, \quad x \in \Omega, t > 0 \\ \qquad u(x,t) = 0, \quad x \in \partial\Omega, t \geq 0 \\ \qquad u(x,0) = u_0(x), u_t(x,0) = u_1(x), \quad x \in \Omega \\ \qquad u_t(x,t-\tau) = f_0(x,t-\tau), \qquad x \in \Omega, t \in (0,\tau) \end{cases}$$

where Ω is a regular and bounded domain of $\mathbb{R}^n$ $(n \geq 1)$, g is a positive real valued decreasing function, $\tau > 0$ represents the delay, μ_1 and μ_2 are positive constants, u_0 , u_1, f_0 are given functions belonging to suitable spaces, and $\partial_t^{\alpha,\beta}$ is the generalized Caputo's fractional derivative defined by:

$$\partial_t^{\alpha,\eta} u(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-s)^{-\alpha} e^{-\eta(t-s)} u_s(s) ds, \quad 0 < \alpha < 1, \quad \eta \geq 0.$$

Mine Result We establish the following two theorems:

Theorem 1. *Assume that* $\beta^{\alpha-1}\mu_2 \leq \sqrt{1-d}\mu_1$. *Then given* $u_0 \in H_0^1(\Omega)$, $u_1 \in L^2(\Omega)$, $f_0 \in L^2(\Omega \times (0,1))$ *and* $T > 0$, *there exists a unique weak solution* (u,z) *of the problem (1)on* $(0,T)$ *such that*

$$u \in C([0,T], H_0^1(\Omega)) \cap C^1([0,T], L^2(\Omega)),$$
$$u_t \in L^2(0,T; H_0^1(\Omega)) \cap L^2((0,T) \times \Omega).$$

Theorem 2. *Assume that* $\beta^{\alpha-1}\mu_2 \leq \sqrt{1-d}\mu_1$. *Then Then, there exist two positive constants* K_1 *and* K_2 *independent of* t *such that for any solution of problem (1), we have*

$$E(t) \leq K_1 e^{-K_2 t}, \quad \forall t \geq 0.$$

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