A reliable analytical technique for solving nonlinear Caputo time-fractional gas dynamics equations

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This study presents a new method called Shehu decomposition method (SDM) for solving nonlinear time-fractional gas dynamics equations. Fractional derivatives are considered in the Caputo sense. To validate the efficiency and reliability of the proposed method, two numerical examples of the nonlinear time fractional gas dynamics equations are considered. The main advantage of SDM is its ease of implementation and its small computational size. Therefore, it is a very effective and efficient semi-analytical method for solving nonlinear fractional partial differential equations.

Consider the nonlinear time-fractional gas dynamics equation

$$D_t^\alpha u + uu_x - u(1-u) = 0, \tag{1}$$

subject to the initial condition

$$u(x,0) = f(x), \tag{2}$$

where $u = \{u(x,t), x \in \mathbb{R}, t \geq 0\}$ and D_t^α is the Caputo time-fractional derivative operator of order α with $0 < \alpha \leq 1$ defined as [3]

$$D_t^\alpha u(x,t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\xi)^{n-\alpha-1} u^{(n)}(x,\xi) d\xi.$$

When $\alpha = 1$, equation (1) reduces into the classical gas dynamics equation. Gas dynamics equations are mathematical expressions based on the physical laws of conservation of mass, conservation of momentum, conservation of energy, etc. The nonlinear fractional gas dynamics equations are applicable in the shock fronts, rare factions, and contact discontinuities [4].

Theorem 1. *Consider the following nonlinear time-fractional gas dynamics equation (1) subject to the initial condition (2), then, the SDM gives the solution of equations (1) and (2) in the form of infinite series which converges rapidly to the exact solution as follows*

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t),$$

where

$$u_n = f(x) - \mathbb{S}^{-1}\left[\frac{v^\alpha}{s^\alpha}(A_n + B_n - u)\right],$$

and $\mathbb{S}$ denotes the Shehu transform [2] of the function u, A_n and B_n are the Adomian polynomials [5] which represent the nonlinear terms uu_x and u^2, respectively, and it can be calculated by the formula below

$$A_n = B_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{i=0}^{\infty} \lambda^i u_i\right)\right]_{\lambda=0}, \quad n = 0, 1, 2, \ldots$$

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