A feasible interior-point algorithm based on a kernel function with a hyperbolic barrier term

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In this work, we present a new primal-dual kernel-based interior-point algorithm for solving linear optimization problems. Our algorithm uses a hyperbolic kernel function to determine the search direction and to measure the distance between the current iterate and the $\mu-$center for the algorithm. We study some properties of the kernel function including e-convexity. After that, we derive the iteration complexity of our algorithm for both large- and small-update methods. Finally, we provide some numerical results to show the practical performance of the algorithm in comparison with other existing kernel-based interior-point algorithms.

We are concerned with the standard linear optimization problem defined as follows:

$$(P)\begin{cases} \min c^T x \\ Ax = b, \\ x \geq 0, \end{cases}$$

where $A \in \mathbb{R}^{m\times n}$, $c \in \mathbb{R}^n$ and $b \in \mathbb{R}^m$ are given, $\operatorname{rank}(A) = m \leq n$ and $x \in \mathbb{R}^n$ is the vector of variables. The dual problem of (P) is given by

$$(D)\begin{cases} \max b^T y \\ A^T y + s = c, \\ s \geq 0, \end{cases}$$

where $y \in \mathbb{R}^m$ and $s \in \mathbb{R}^n$ are the vectors of variables. Following the basics of interior-point methods (IPMs), we arrive at the following system

$$(1) \quad \begin{cases} A\Delta x = 0, \\ A^T\Delta y + \Delta s = 0, \\ s\Delta x + x\Delta s = \mu e - xs. \end{cases}$$

By taking a step along the search direction, one constructs a new iterate point

$$x_+ := x + \alpha\Delta x, \quad y_+ := y + \alpha\Delta y, \quad s_+ := s + \alpha\Delta s,$$

for some $0 < \alpha \leq 1$ satisfying $(x_+, s_+) > 0$.
Defining the scaled vector v and the scaled search directions d_x and d_s as follows

$$v = \sqrt{\frac{xs}{\mu}}, \quad d_x = \frac{v\Delta x}{x}, \quad d_s = \frac{v\Delta s}{s},$$

we rewrite system (2) as follows

$$(2) \quad \begin{cases} \overline{A}d_x = 0, \\ \overline{A}^T\Delta y + d_s = 0, \\ d_x + d_s = v^{-1} - v, \end{cases}$$

where $\overline{A} = \frac{1}{\mu} A V^{-1} X, \;\; V = \operatorname{diag}(v), \;\; X = \operatorname{diag}(x)$.
A crucial observation is that the right-hand side in the last equation of system (2) is equal to minus gradient of the classical logarithmic scaled barrier (proximity) function

$$\Psi(v) = \sum_{i=1}^{n} \psi_c(v_i),$$

where

$$\psi_c(t) = \frac{t^2 - 1}{2} - \log t.$$

Following the basic of kernel-based primal-dual IPMs, we replace ψ_c by a new kernel function. The latter belongs to the newly developed hyperbolic type [2,3].

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