TIMOSHENKO SYSTEM OF SECOND SOUND WITH TIME-VARYING DELAY: WELL POSEDNESS AND STABILITY

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In this work, we study the well-posedness and exponential stability of thermoelastic system of Timoshenko type with a time-varing delay. We show the well-posedness using the semi-group theory, and we prove an exponential stability result under the usuel assumption on the wave speed by the energy method (see [1]-[3]).

The aim of this studies is to investigate the effect of time delay on the behavior of the solution for the following system

$$\begin{cases} \rho_1\varphi_{tt}(x,t) - K(\varphi_x+\psi)_x(x,t) + \mu_1\varphi_t(x,t) + \mu_2\varphi_t(x,t-\tau(t)) = 0, \\ \rho_2\psi_{tt}(x,t) - b\psi_{xx}(x,t) + K(\varphi_x+\psi)(x,t) + \gamma\theta_x(x,t) = 0, \\ \rho_3\theta_t(x,t) + \kappa q_x(x,t) + \gamma\psi_{tx}(x,t) = 0, \\ \tau_0 q_t(x,t) + \delta q(x,t) + \kappa\theta_x(x,t) = 0, \end{cases} \tag{1}$$

where the space variable is $x \in (0,1)$ and the time variable is $t \in (0,\infty)$, the functions ψ and φ are the rotation angle and the transverse displacement of the solid elastic material, respectively; the function θ is the temperature difference, $q = q(t,x) \in \mathbb{R}$ is the heat flux, and ρ_1, ρ_2, ρ_3, γ, τ_0, δ, κ, μ_1, μ_2 and K are positive constants and $\tau > 0$ represents the time delay. We consider about the ensuing initial conditions,

$$\begin{cases} \varphi(x,0) = \varphi_0(x), \quad \varphi_t(x,0) = \varphi_1(x), \quad \psi(x,0) = \psi_0(x), \quad \psi_t(x,0) = \psi_1(x), \\ \theta(x,0) = \theta_0(x), \quad q(x,0) = q_0(x), \quad \varphi_t(x,t-\tau(t)) = f_0(x,t-\tau(t)), \end{cases} \tag{2}$$

where $x \in (0,1)$ and $t \in (0,\tau)$. We consider the boundary conditions

$$\varphi(0,t) = \varphi(1,t) = \psi(0,t) = \psi(1,t) = q(0,t) = q(1,t) = 0, \qquad \forall t \geq 0. \tag{3}$$

The problem (1) can therefore be rewritten as follows:

$$\begin{cases} \rho_1\varphi_{tt}(x,t) - K(\varphi_x+\psi)_x(x,t) + \mu_1\varphi_t(x,t) + \mu_2 z(x,1,t) = 0, \\ \rho_2\psi_{tt} - b\psi_{xx} + K(\varphi_x+\psi) + f(\psi) + \gamma\theta_x = 0, \\ \rho_3\theta_t + \kappa q_x + \gamma\psi_{tx} = 0, \\ \tau_0 q_t + \delta q + \kappa\theta_x = 0, \\ \tau(t)z_t(x,\rho,t) + (1-\tau'(t)\rho)z_\rho(x,\rho,t) = 0, \qquad (x,\rho,t) \in (0,1)\times(0,1)\times(0,+\infty), \end{cases} \tag{4}$$

where $x \in (0,1)$, $\rho \in (0,1)$, and $t > 0$. The following initial conditions were applied to the aforementioned system,

$$\left.\begin{aligned} &\varphi(x,0)=\varphi_0(x), \quad \varphi_t(x,0)=\varphi_1(x), \\ &\psi(x,0)=\psi_0(x), \quad \psi_t(x,0)=\psi_1(x), \\ &\theta(x,0)=\theta_0(x), \quad q(x,0)=q_0(x), \end{aligned}\right\} \qquad x\in(0,1)$$
$$\begin{aligned} &z(x,0,t)=\varphi_t(x,t), && x\in(0,1), \quad t>0 \\ &z(x,\rho,0)=f_0(x,1-\rho\tau(0)), && (x,\rho)\in(0,1)\times(0,1). \end{aligned} \tag{5}$$

Where the function $\tau(t)$ satisfies the condition

$$0<\tau_0\leq\tau(t)\leq\overline{\tau}, \forall t>0. \tag{6}$$

$$\tau'(t)\leq d<1, \forall t>0, \tag{7}$$

and

$$\tau\in W^{2,\infty}([0,T])\,\forall T>0 \tag{8}$$

Together with the aforementioned initial conditions, we also take into account the following boundary conditions:

$$\varphi(0,t)=\varphi(1,t)=\psi(0,t)=\psi(1,t)=q(0,t)=q(1,t)=0, \qquad \forall t\geq 0. \tag{9}$$

Theorem 1. *Our existence and uniqueness result reads as follows. Assuming that* (6),(7),(8)*and* $\mu_2<\sqrt{1-d}\mu_1$*, hold, then we have the following results.*
(i) If $U_0\in\mathscr{H}$ *then problem*(1) *has a unique mild solution* $U\in C([0,\infty)\,\mathscr{H})$ *with* $U_0=U(0)$.
(ii) If U_1 *and* U_2 *are two mild solutions of the problem* (1)*, then there exists a positive constant*$C_0=C(U_1(0),U_2(0))$ *such that*

$$||U_1(t)-U_2(t)||_{\mathscr{H}}\leq e^{C_0T}||U_1(0)-U_2(0)||_{\mathscr{H}} \quad \textit{for any } 0\leq t\leq T \tag{10}$$

(iii) If $U_0\in D(\mathscr{A})$*, then the above mild solution is a strong solution.*

Exponential stability for $\mu_2<\sqrt{1-d}\mu_1$:

Theorem 2. *Assuming* (6), (7), (8) *and* $\mu_2<\sqrt{1-d}\mu_1$*, two positive constants* C *and* γ*, that are independent of* t*, exist, and for any solution of the problem* (4)$-$(9)*, we have*

$$E(t)\leq Ce^{-\gamma t}, \qquad \forall t\geq 0. \tag{11}$$

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