The exact solution of an axisymmetric problem for a semi-infinite poroelastic cylinder

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The semi-infinite poroelastic cylinder $(0 < r < a, 0 < z < \infty, -\pi < \varphi < \pi)$ is considered in terms of Biot's model [1]. The cylindrical surface $r = a$ is loaded by mechanical and fluid pressure loads

$$\sigma_r(a, z) = -L(z), \tau_{rz}(a, z) = T(z), p(a, z) = P(z), \tag{1}$$

where $\sigma_r(r, z), \tau_{rz}(r, z)$ are normal and tangential stress respectively, $p(r, z)$ is pore pressure, $L(z), T(z), P(z)$ are known functions. The upper boundary $z = 0$ is in ideal contact and impermeable conditions [2]

$$w(r, 0) = 0, \tau_{rz}(r, 0) = 0, \frac{\partial p}{\partial z}(r, 0) = 0, \tag{2}$$

where $w(r, z) = u_z(r, z)$ is displacement function. The displacements, stress and pore pressure that satisfy conditions (1)-(2) and equilibrium and storage equations [2] should be found.

The original problem is reduced to the one-dimensional problem with the help of semi-infinite sin-, cos- Fourier transforms applied regarding variable z. The one-dimensional boundary value problem is rewritten as vector boundary value problem

$$\begin{gathered} L_2 \vec{y}_\beta(r) = 0, 0 < r < a, \\ A_\beta \vec{y}_\beta'(a) + B_\beta \vec{y}_\beta(a) = \vec{g}_\beta. \end{gathered} \tag{3}$$

Here L_2 is differential operator of the second order, $\vec{y}_\beta(r) = (u_\beta, w_\beta, p_\beta)^T$ is the vector with displacement and pore pressure transforms, A_β, B_β are known matrices, $\vec{g}_\beta$ is known vector.

The solution of the vector boundary value problem (3) is constructed with the help of matrix differential calculation apparatus [3]. The analytical expressions for the displacements and pore pressure transforms are derived in the explicit form

$$\vec{y}_\beta(r) = (Y_1(r) + Y_3(r)) (c_1, c_2, c_3)^T, \tag{4}$$

where $Y_1(r), Y_3(r)$ are fundamental matrix solutions of the matrix differential equation corresponding to (4), $c_i, i = \overline{1,3}$ are constants that are found from the boundary conditions in (3).

The application of inverse Fourier transform to (4) finishes the construction of exact solution of the original problem. The stress and pore pressure were investigated regarding the change of cylinder's radius and the poroelastic material.

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