INTERIOR POINT METHOD FOR CONVEX QUADRATIC PROGRAMMING

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As we know, quadratic programming (QP) is minimizing or maximizing an objective function subject to bounds, linear equality, and inequality constraints. The difficulty of solving the (QP) problem depends largely on the nature of the matrix Q. So In order to solve a quadratic programming problem many researchers want to find a new method which is easy and low cost. Therefore, we provide a logarithmic barrier interior point method without line search [1,3] for resolving convex quadratic programming issue with inequality constraints. This method based on the idea that the minorant function approximates the barrier function. The benefit of these functions is that they make the calcul of the displacement step simple and quick, contrarily to the line search method, which is time-consuming and expensive to identify the displacement step. We consider the following quadratic programming problem

$$(QP)\begin{cases}\min \ \ q(x) & = \frac{1}{2}x^tQx + c^tx \\ Ax \geq b & x \in \mathbb{R}^m\end{cases} \tag{1}$$

Where Q is a $\mathbb{R}^{n\times n}$ symmetric semidefinite matrix, $A \in \mathbb{R}^{m\times n}$, such that $rangA = m < n$, $c \in \mathbb{R}^n$ and $b \in \mathbb{R}^m$.

The problem (QP) is approximated by the following perturbed problem (D_η)

$$(D_\eta)\begin{cases}\min \ q_\eta(x) \\ x \in \mathbb{R}^m,\end{cases} \tag{2}$$

with the penalty parameter $\eta > 0$, and f_η is the barrier function defined by

$$q_\eta(x) = \begin{cases} q(x) - \eta\sum\limits_{i=1}^{m} \ln\langle e_i,\ Ax-b\rangle & if \quad A^Tx - b > 0 \\ +\infty & otherwise.\end{cases} \tag{3}$$

where $(e_1, e_2, ..., e_n)$ is the canonical base in $\mathbb{R}^n$. We are interested then in solving the problem (D_η). We know that if the matrix $H = \nabla^2 q_\eta(x)$ is positive definite, then the problem (D_η) is strictly convex, if it has a solution it is unique. to solve (D_η) we need to calculate the displacement step α_k in the following recursive formula

$$x_{k+1} = x_k + \alpha_k d_k. \tag{4}$$

where d_k is the descent direction, we used the Newton direction defined by:

$$\nabla^2 q_\eta(x)d_k = -\nabla q_\eta(x). \tag{5}$$

in this work, we are interested in the theoretical performances of the interior point barrier logarithmic method, we study more closely the effective calculation of the step of displacement by a new technique concerning the minorant function [2]. We approximate the function: $\varphi(\alpha) = \frac{1}{\eta}\left(q_\eta(x+\alpha d) - q_\eta(x)\right)$. by simple minorant function. We can show that:

$$\varphi(\alpha) = n\left(\sum_{i=1}^{n} z_i\right)\alpha - \|z\|^2\alpha - \sum_{i=1}^{n}\ln(1+z_i\alpha) + \frac{1}{2\eta}\alpha^2 d^t Q d - \frac{1}{\eta}\alpha d^t Q d,\ \alpha \in [0,\widehat{\alpha}]\,.$$

Such that $z_i = \frac{\langle e_i, Ad\rangle}{\langle e_i, Ax-b\rangle}$, $\varphi(\alpha)$ verifies the following properties :

$$\|z\|^2 = n(\bar{z}^2 + \sigma_z^2) = \varphi''(0) = -\varphi'(0),\ \varphi(0) = 0.$$

In our case, we take $x_i = 1 + tz_i$, so we have $\bar{x} = 1 + t\bar{z}$ and $\sigma_x = \alpha\sigma_z$. Which gives us the lower bound function, then: $\widetilde{\varphi}_0(\alpha) = \frac{\|z\|^2}{\beta_0}\alpha - k\ln(1 + \frac{\|z\|^2}{\beta_0}\alpha) - \frac{1}{r}\widehat{\alpha} d^t Q d$. Such as : $\beta_0 = \bar{z} + \sigma_z\sqrt{n-1}$ The logarithms are well defined as soon as $\alpha \leq \widehat{\alpha}\widehat{\alpha}_0 : \widetilde{\varphi}_0(\alpha) \geq \varphi(\alpha)$. With $\widetilde{\varphi}_0$ is convex and satisfies the conditions.

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