# Study of a competition model between two species of microorganisms with interaction intraspecific in a chemostat

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In this work, we will study a competition model of two species of microorganisms with intraspecific interaction between species themselves on a complementary substrate in a chemostat. To understand the asymptotic behavior and the global analysis of this model, we divide our work into two parts:
- The first part is a general introduction to the chemostat and to mathematical models, we introduce the one-species mathematical model (simple chemostat) and the to two species (chemostat with competition) by differential equations of biological processes that we use and study later.
- In the second part, we present the model of competition between two species of microorganisms with intraspecific interaction in a chemostat. The model is presented in the form of a system of nonlinear ordinary differential equations. This model is written:

$$\begin{cases} \dfrac{ds}{dt} = (s^0 - s)D - \dfrac{f_1(s)}{\gamma_1}x_1 - \dfrac{f_2(s)}{\gamma_2}x_2 \\ \dfrac{dx_1}{dt} = (-D_1 + f_1(s) - q_{11}(x_1))x_1 \\ \dfrac{dx_2}{dt} = (-D_2 + f_2(s) - q_{22}(x_2))x_2 \\ s(0) \geq 0 \;\;,\;\; x_i(0) \geq 0 \;\;,\;\; i = 1, 2. \end{cases} \tag{1}$$

Or:
$s$, $s^0$, $x_1$ and $x_2$ are respectively the concentrations of the substrate, substrate at the inlet of the chemostat and the microorganisms.
$f_1$ and $f_2$ are microorganism growth functions.
$\gamma_1$ and $\gamma_2$ are the yield constants of $x_1$ and $x_2$ respectively.
$q_{11}$ and $q_{22}$ describes intraspecific interference between the two species.
$D$, $D_1$ and $D_2$ are respectively the dilution rates of $s$, $x_1$ and $x_2$.

We globally analyze the model with the general growth function using the convergence theorem of *Thieme* after performing the dimensional reduction.
-Finally, we do numerical simulations to clearly see the stability of the equilibrium points.

We consider the following dynamic system:

$$\dot{x} = f(x)\ ,\ x(t_0) = x_0 \tag{2}$$

where f is a function defined on a open $\Omega \in R^n$ to values in $R^n$ and class $C_1$.

**Theorem 1.** *(Thieme-Zhao) [5]*
*- Let $X$ be a closed subset of $R^n_+$. We consider the dissipative system (2). We assume that $X$ is positively invariant for this system.*
*- Let $X = X_1 \cup X_2,\ X_1 \cap X_2 = \emptyset$ where: $X_1$ a positively invariant subset and $X_2$ is a closed*

*subset of $R^n_+$.*
*- Let $X$ be a finite set of equilibrium points of the system (2) in $X_2$, we assume that:*
*1 - Any solution of the system (2) resulting from $X_2$ and which remains in $X_2$ for all t converges to an equilibrium point of X.*
*2 - Any equilibrium point of M is an isolated invariant set in X and it is weakly repulsive for $X_1$.*
*3 - M is acyclic in $X_2$ .*
*- Then $X_2$ is uniformly repulsive for $X_1$. Also, if $X_1$ is convex, there is at least one equilibrium point in $X_1$.*

**Definition 1.** Let $X = X_1 \cup X_2, X_1 \cap X_2 = \emptyset$ and $Y_2 \in X_2$, or $X_2$ is a closed subset of $R^n_+$ and $X_1$ a positively invariant subset. We have the following definitions:
1. $Y_2$ is said to be weakly repulsive for $X_1$ if: $\limsup_{t\to+\infty} d(x(t), Y_2) > 0$
2. $Y_2$ is said to be repellent for $X_1$ if: $\liminf_{t\to+\infty} d(x(t), Y_2) > 0$
3. $Y_2$ is said to be uniformly repulsive for $X_1$ if there exists a constant $\epsilon > 0$ such that: $\liminf_{t\to+\infty} d(x(t), Y_2) > \epsilon$
where x(t) the solution of the system (2) with $x(0) \in X_1$ and d denotes the usual distance in $R^n$.

**Remark 1.** The system (2) is said to be:
(i) weakly persistent if and only if the boundary of X is weakly repelling.
(ii) persistent if and only if the boundary of X is repulsive.
(iii) uniformly persistent if and only if the boundary of X is uniformly repulsive.

According to the competition between these two species of micro-organisms with the interaction intraspecific in a chemostat it is concluded that the species which has the smallest threshold profitability prevails over the other. This result is called the principle of competitive exclusion, which is in contradiction with the natural environment where two species can coexist in the long term.

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