Euler Elastic Model & Its Treatment

Muhammad Sami Siddiqui
Institute of Business Administration, Karachi, Pakistan
ssiddiqui@iba.edu.pk

Elastic body deformation at different loading conditions has various applications in engineering, soft structures, and soft robotics. It also serves medical purposes, such as comprehending the forms of vesicles and cell membranes and their deformation behavior.

In this talk, we will consider the Euler elastica ring model subjected to uniform external force p under various loading conditions [1].

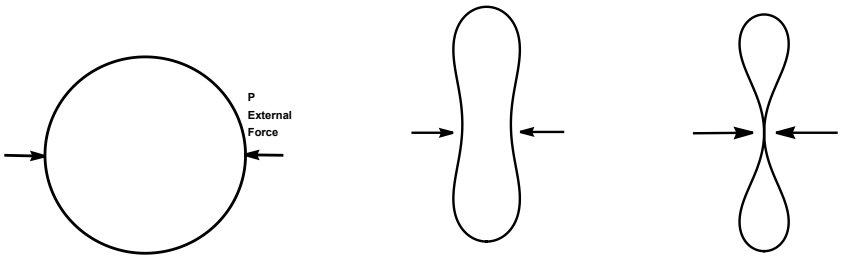

Figure 1: Elastic ring deforms under two-sided symmetric uniform external force p

$$v''(s) + \mu v(s) + \frac{3}{2}v(s)^2 + \frac{1}{2}v(s)^3 - \beta = 0. \tag{1}$$

We will see the solution of Euler elastica equation (1), curvature-based nonlinear differential equation, through different approximation methods [2] and show that harmonic balance works better compared with the analytical approach (Elliptic Function) [3]. Furthermore, we will also discuss the stability diagram of the elastic ring using harmonic balance formulation over the range of p. Finally, some asymmetric cases of an elastic ring will be discussed, and its solution through harmonic balance method.

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3. Djondjorov P. A. et al. Analytic description and explicit parametrisation of the equilibrium shapes of elastic rings and tubes under uniform hydrostatic pressure. International Journal of Mechanical Sciences, 2011, Vol. 53, 355-364.