A Derivative-Free algorithm for continuous global optimization

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In this work, we consider the bound constrained global optimization problem of the following form:

$$f^* = \min_{x\in D} f(x). \tag{1}$$

where $D = \prod_{i=1}^{n} [L_i, U_i] \subset \mathbb{R}^n$ with L_i, U_i are real numbers for $i = 1, \dots, n$ and the real objective function $f(x)$ is only continuous. The problem (1) is of interest in many real-world problems involving objective functions which are only continuous and do not possess strong mathematical properties (such as convexity, differentiability, Lipschitz or Hölderian continuity etc.). If a function f is a priori known to be only continuous, then apart from saying that f "remains near $f(x)$ in a neighbourhood of x" which translates into dealing with the modulus of continuity of f, at x nothing tractable can be inferred on the values of f away from x. One way to tackle such enormous uncertainties is to use the following result:

Theorem 1. *[3] Let f be a real function defined on a compact set $D \subset \mathbb{R}^n$. Then f is continuous if and only if for all $\varepsilon > 0$, there exists a constant $C_\varepsilon > 0$ such that for all $x, x' \in D$, we have:*

$$|f(x) - f(x')| \le C_\varepsilon \|x - x'\| + \varepsilon \tag{2}$$

A drawback of the above theorem is that given f and ε, there are no specific means to recover exactly the constant C_ε for example for black-box functions. Our idea is to work on a sequence $\{C_j\}_{j\in\mathbb{N}}$ of positive constants that controls the growth of C_ε. This is done by actual collecting of information while running a Lissajous curve throughout the feasible domain.

The proposed method is based on the reducing transformation technique by running in the feasible domain a single parametrized Lissajous curve, which becomes increasingly denser and progressively fills the feasible domain. By means of a one-dimensional search algorithm, we realize a mixed method which explores the feasible domain. To speed up the mixed exploration algorithm, we have incorporated a derivative-free local search algorithm to explore promising regions. This method converges in a finite number of iterations to the global minimum. The simulation results based on a set of 180 benchmark functions with diverse properties and different dimensions show the efficiency and the abilities of the proposed algorithm in finding the global optima compared with the existing methods.

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