Average no-regret & average low-regret controls for a thermoelastic system

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This work deals with the average optimal control of a thermoelastic system depending on a coupled parameter, with missing initial conditions. We broach the concept of average no-regret control and its approach, the average low-regret control to get a general description from our optimal control to an optimality system, by using the Euler Lagrange first-order optimality condition.

Let Ω be an open bounded of $\mathbb{R}^3$, smooth boundary Γ of class $C^2, \Gamma_0 \subset \Gamma$, denote by $\Sigma = (0,T)\times\Gamma$, $Q = (0,T)\times\Omega, \omega$ is a non-empty bounded subset of Ω, For $T > 0$, the following coupled systems with missing initial conditions, are considered by [1]

$$\begin{cases} y_{tt} - \mu\Delta y - (\lambda+\mu)\nabla divy + \alpha\nabla\theta = f + v\chi_\omega & \text{in } Q, \\ \theta_t - \Delta\theta + \alpha div y_t = 0 & \text{in } Q, \\ y(x,0) = y_0(x), y_t(x,0) = y_1(x), & \\ \theta(x,0) = \theta_0(x) & \text{in } \Omega, \\ y = \theta = 0 & \text{on } \Sigma, \end{cases} \tag{1}$$

The coupled systems (1) are used to define the homogeneous isotropic thermoelastic body elastic and thermal behavior, wherever, $\lambda, \mu > 0$ are the Lame constants, χ_ω is the characteristic function of ω, $y = (y_1, y_2, y_3)$ is the vector of the elastic displacement at the moment t from $x = (x_1, x_2, x_3)$, the elastic temperature $\theta = \theta(x,t)$ is a scalar function, α is the coupling parameter in $(0,1)$, $g = (y_0, y_1, \theta_0)$ are the initial conditions expected to be unknown in $H = (H_0^1(\Omega))^3 \times (L^2(\Omega))^3 \times L^2(\Omega)$, $f \in L^2\left(0,T;(H_0^1(\Omega))^3\right)$ and, $v = (v_1, v_2, v_3)$ is a distributed control vector with an external force in U_{ad}, $U_{ad} = \{v \in (L^2(Q))^3 : v_{\min} \le v \le v_{\max}\}$ is a convex, closed and non-empty subset, $v_{\max}$, $v_{\min}$ are the maximum and minimum of permitted force values to reserve the elasticity property of the considered body. The next couple present the solution of the system (1).

$$\begin{aligned} &y(v,g,\alpha) \in C\left(0,T;(H_0^1(\Omega))^3\right) \cap C^1\left(0,T;(L^2(\Omega))^3\right), \\ &\theta(v,g,\alpha) \in C\left(0,T;L^2(\Omega)\right). \end{aligned}$$

Associate to (1) the following quadratic cost function [2], [5]:

$$J(v,g) = \left\|\int_0^1 y(v,g,\alpha)\,d\alpha - y_d\right\|^2_{L^2\left(0,T;\left(H_0^1(\Omega)\right)^3\right)} + \left\|\int_0^1 \theta(v,g,\alpha)\,d\alpha - \theta_d\right\|^2_{L^2(0,T;L^2(\Omega))} + \beta\|v\|^2_{(L^2((0,T)\times\omega))^3}, \tag{2}$$

where θ_d, y_d are given observations of deformation and temperature respectively. with $\beta > 0$.

The next coupled systems presents the characterization of the average no-regret control u solution of $(1)-(2)$:

$$\begin{cases} y_{tt} - \mu\Delta y - (\lambda+\mu)\nabla.divy + \alpha\nabla\theta = f + u\chi_\omega, \\ \theta_t - \Delta\theta + \alpha div y_t = 0, \\ y(x,0) = 0, y_t(x,0) = 0, \theta(x,0) = 0, \\ y = \theta = 0. \end{cases}$$

$$\begin{cases} \varphi_{tt} - \mu\Delta\varphi - (\lambda+\mu)\nabla.div\varphi + \alpha\nabla\psi_t = \int_0^1 [y(u,0,\alpha) - y(0,0,\alpha)]\,d\alpha, \\ -\psi_t - \Delta\psi - \alpha div\varphi = \int_0^1 [\theta(0,0,\alpha) - \theta(0,0,\alpha)]\,d\alpha, \\ \varphi(x,T) = 0, \varphi_t(x,T) = 0, \psi(x,T) = 0, \\ \varphi = \psi = 0. \end{cases}$$

$$\begin{cases} \rho_{tt} - \mu\Delta\rho - (\lambda+\mu)\nabla.div\rho + \alpha div\sigma = 0, \\ \sigma_t - \Delta\sigma + \alpha\nabla\rho_t = 0, \\ \rho(x,0) = \frac{1}{\gamma}\left(\int_0^1 \alpha\nabla\psi(x,0)\,d\alpha - \int_0^1 \varphi_t(x,0)\,d\alpha\right), \\ \rho_t(x,0) = \frac{1}{\gamma}\int_0^1 \varphi(x,0)\,d\alpha, \\ \sigma(x,0) = \frac{1}{\gamma}\int_0^1 \psi(x,0)\,d\alpha \\ \rho = \sigma = 0. \end{cases}$$

$$\begin{cases} p_{tt} - \mu\Delta p - (\lambda+\mu)\nabla.divp + \alpha divq_t = \int_0^1 [y(u,0,\alpha) - y(0,0,\alpha)]\,d\alpha - y_d + \int_0^1 \rho d\alpha, \\ -q_t - \Delta q - \alpha\nabla p = -\theta_d + \int_0^1 \sigma d\alpha + \int_0^1 [\theta(u,0,\alpha) - \theta(0,0,\alpha)]\,d\alpha \text{ in } Q, \\ p(x,T) = 0, p_t(x,T) = 0, q(x,T) = 0, \\ p = q = 0. \end{cases}$$

With the average optimality condition :

$$\int_0^T \int_\omega \left(\int_0^1 pd\alpha + \beta u\right)(v-u)\,dxdt \geq 0, \ \forall v \in U_{ad}.$$

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