Synchronization between fractional-order chaotic systems and integer order chaotic systems (fractional-order chaotic systems)

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Based on the idea of tracking control and stability theory of fractional-order systems, a controller is designed to synchronize the fractional-order chaotic system with chaotic systems of integer orders, and synchronize the different fractional-order chaotic systems. The proposed synchronization approach in this work shows that the synchronization between fractional-order chaotic systems and chaotic systems of integer orders can be achieved, and the synchronization between different fractional-order chaotic systems can also be realized. Numerical experiments show that the present method works very well.

In recent years, the fractional calculus has become an excellent tool in modeling many physical phenomena and engineering problems. One of the very important areas of application of fractional calculus is chaos theory. Chaos is a very interesting nonlinear phenomenon that has been intensively studied over the past two decades [1]. The chaos theory is found to be useful in many areas such as data encryption, financial systems, biology and biomedical engineering, etc. Fractional-order chaotic dynamical systems have begun to attract a lot of attention in recent years and can be seen as a generalization of chaotic dynamic integer-order systems. The synchronization between the fractional-order chaotic system and the integer-order chaotic system is thoroughly a new domain and began to attract much attention in recent years because of its potential applications in secure communication and cryptography. Then the idea of the synchronization is to use the output of the master (drive) system to control the slave (response) system so that the output of the slave system tracks asymptotically the output of the master system. In the past twenty years, various types of synchronization have been proposed and investigated, e.g., complete synchronization [2], lag synchronization, phase synchronization, project synchronization, generalized synchronization, etc. As a special case of generalized synchronization, anti-synchronization is achieved when the sum of the states of master and slave systems converges to zero asymptotically with time. In this research work,we apply nonlinear control theory to synchronize two chaotic systems when an fractional- order system is chosen as the drive system and a integer-order system serves as the response system and synchronize two identical fractional-order chaotic systems, we demonstrate the technique capability on the synchronization between fractional-order lesser date moth chaotic system and integer-order chaotic system (fractional-order chaotic systems).

Consider the following fractional-order chaotic system as a drive (master) system

$$D^{\alpha}x_1 = Ax_1 + g(x_1), \tag{1}$$

where $x_1 \in \mathbb{R}^n$ is the state vector, $A \in \mathbb{R}^{n\times n}$ is the linear part, $g(x_1)$ is a continuous nonlinear function, and D^{α} is the Caputo fractional derivative.
Also, the response system (slave) can be described as

$$\dot{x}_2 = Ax_2 + g(x_2) + u(t), \tag{2}$$

where $x_2 \in \mathbb{R}^n$ is the state vector, $A \in \mathbb{R}^{n\times n}$ is the linear part, and $g(x_2)$ is a continuous nonlinear function and $u(t) \in \mathbb{R}^n$ is the control.
Define the synchronous errors as $e = x_2 - x_1$.
Our aim is to determine the controller $u(t) \in \mathbb{R}^n$ such that the drive system and response system are synchronized (i.e., $\lim_{t\to\infty} \|e(t)\| = 0$).
The synchronisation error system between the driving system (1) and the response system (2) can be expressed as

$$\dot{e} = \dot{x}_2 - \dot{x}_1,$$

where $\dot{x}_2$ is obtained from the response system (2), while no exact expressions of $\dot{x}_1$ can be obtained from the driving system (1). Therefore, the numerical differentiation method is used to obtain $\dot{x}_1$. According to the definition of derivative, the derivative is approximately expressed using the difference quotient as

$$g'(a) \approx \frac{g(a+h) - g(a)}{h} \tag{3}$$

$$g'(a) \approx \frac{g(a) - g(a-h)}{h}, \tag{4}$$

where $(h > 0)$ is a small increment.

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