NUMERICAL STUDY FOR AN INTEGRO-DIFFERENTIAL NONLINEAR VOLTERRA EQUATION

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This work deals with the numerical solution of nonlinear Volterra integro-differential equations by spline collocation. We applied the iterative collocation method to obtain an approximate solution without needed to solve any algebraic system.

We investigate an iterative collocation method for the following Volterra integro-differential equation

$$x'(t) = f(t) + Q(t, x(t)) + \int_0^t K(t, s, x(s))ds, x(t_0) = x_0, t \in I = [0, T], \tag{1}$$

where the functions f, Q and K are sufficiently smooth.There are several numerical methods for approximating the solution of equation (1). For example, spectral methods, Galerkin methods, collocation methods, and Legendre wavelets series, (cf, e.g. [1-4], and references therein).

Let Π_N be a uniform partition of the interval $I = [0, T]$ defined by $t_n = nh, \quad n = 0, ..., N-1$, where the stepsize is given by $\frac{T}{N} = h$. Let the collocation parameters be $0 \le c_1 < ...... < c_m \le 1$ and the collocation points be $t_{n,j} = t_n + c_j h$, $j = 1, ..., m, n = 0, ..., N-1$. Define the subintervals $\sigma_n = [t_n, t_{n+1}]$, and $\sigma_{N-1} = [t_{N-1}, t_N]$. Moreover, denote by π_{m+1} the set of all real polynomials of degree not exceeding $m+1$.
We define the real polynomial spline space of degree $m+1$ as follows:

$$S^{(1)}_{m+1}(\Pi_N) = \{u \in C^1(\mathrm{I}, \mathbb{R}) : u_n = u/\sigma_n \in \pi_{m+1}, n = 0, ..., N-1\}.$$

It holds for any $u \in S^1_{m+1}(I, \Pi_N)$ that

$$u_n(t_n + sh) = u_{n-1}(t_n) + hB_0(s)u'_{n-1}(t_n) + h\sum_{j=1}^{m} B_j(s)u'_n(t_{n,j}), s \in [0, 1]. \tag{2}$$

Now, we approximate x by $u \in S^1_{m+1}(I, \Pi_N)$ such that $u'(t_{n,j})$ satisfy the following nonlinear system,

$$\begin{aligned} u'_n(t_{n,j}) &= f(t_{n,j}) + Q(t_{n,j}, u(t_{n,j})) + h\sum_{p=0}^{n-1} K(t_{n,j}, t_p, u_p(t_p)) + h^2 \sum_{p=0}^{n-1} b_0 K'(t_{n,j}, t_p, u_p(t_p)) \\ &+ h^2 \sum_{p=0}^{n-1}\sum_{v=1}^{m} b_v K'(t_{n,j}, t_{p,v}, u_p(t_{p,v})) + hc_j K(t_{n,j}, t_n, u_{n-1}(t_n)) \\ &+ h^2 a_{j,0} K'(t_{n,j}, t_n, u_{n-1}(t_n)) + h^2 \sum_{v=1}^{m} a_{j,v} K'(t_{n,j}, t_{n,v}, u_n(t_{n,v})), \end{aligned} \tag{3}$$

for $n = 0, ..., N-1$, $j = 1, ..., m$ where $u'_{-1}(t_0) = x'(0) = f(0)$ and $u_{-1}(t_0) = x(0)$.

In our convergence analysis, we study the following linear Volterra integro-differential equation

$$x'(t) = f(t) + Q(t, x(t)) + \int_0^t K(t,s)x(s)ds, t \in I = [0,T], \tag{4}$$

The following result gives the existence and the uniqueness of a solution for the linear system.

Lemma 1. *For sufficiently small h, the linear system (4) defines a unique solution $u \in S^1_{m+1}(I, \Pi_N)$ which is given by (2).*

The following result gives the convergence of the approximate solution u to the exact solution x.

Theorem 1. *Let f, K be $m+2$ times continuously differentiable on their respective domains. If $-1 < R(\infty) = (-1)^m \prod_{l=1}^{m} \frac{1-c_l}{c_l} < 1$, then, for sufficiently small h, the collocation solution u converges to the exact solution x, and the resulting errors functions $e^{(v)} := x^{(v)} - u^{(v)}$ for $v = 0, 1$ satisfies:*

$$\|e^{(v)}\|_{L^\infty(I)} \leq Ch^{m+1},$$

for $v = 0, 1$ and C is a finite constant independent of h.

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