Analysis of a dynamic viscoelastic frictionless contact problem with adhesion

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We study a frictionless contact problem between a viscoelastic body and an obstacle. The process is assumed to be dynamic and the contact is modelled with a version of normal compliance and adhesion. We derive a variational formulation for the problem then we prove the existence and the uniqueness of a weak solution. The proof is based on nonlinear evolution equations, monotone operators theory and fixed point arguments, see for instance [1]-[3].

Let's consider the following adhesive dynamic contact problem

Problem P

Find a displacement field $u : \Omega \times [0,T] \to \mathbb{R}^d$ and a stress field $\sigma : \Omega \times [0,T] \to S^d$ and a bonding field $\beta : \Omega \times [0,T] \to [0,1]$ such that

$$\begin{array}{lll}
\sigma = A\varepsilon(\dot{u}) + G\varepsilon(u) & \text{in} & \Omega \times (0,T), \\
\rho\ddot{u} = Div\sigma + f_0 & \text{in} & \Omega \times (0,T), \\
u = 0 & \text{on} & \Gamma_1 \times (0,T), \\
\sigma\nu = f_2 & \text{on} & \Gamma_2 \times (0,T), \\
-\sigma_\nu = p_\nu(u_\nu) - \gamma_\nu\beta^2 R_\nu(u_\nu) & \text{on} & \Gamma_3 \times (0,T), \\
-\sigma_\tau = p_\tau(\beta)R_\tau(u_\tau) & \text{on} & \Gamma_3 \times (0,T), \\
\dot{\beta} = -\left(\beta\left(\gamma_\nu(R_\nu(u_\nu))^2 + \gamma_\tau\|R_\tau(u_\tau)\|^2\right) - \varepsilon_a\right)_+ & \text{in} & \Gamma_3 \times (0,T), \\
\beta(0) = \beta_0 & \text{in} & \Gamma_3, \\
u(0) = u_0, \dot{u}(0) = v_0 & \text{in} & \Omega.
\end{array} \tag{1}$$

Using Green's formula, we get

Problem PV

Find a displacement field $u : [0,T] \to V$ and a stress field $\sigma : [0,T] \to \mathcal{H}$ and a bonding field $\beta : [0,T] \to L^\infty(\Gamma_3)$ such that

$$\sigma(t) = A\varepsilon(\dot{u}) + G\varepsilon(u(t)), \tag{2}$$

$$(\ddot{u}, v)_{V'\times V} + (\sigma, \varepsilon(v))_{V'\times V} + j_{ad}(\beta, u, v) = (f(t), v)_{V'\times V}\ \forall v \in V,\ \text{a.e. } t \in (0,T), \tag{3}$$

$$\dot{\beta} = -\left(\beta\left(\gamma_\nu(R_\nu(u_\nu))^2 + \gamma_\tau\|R_\tau(u_\tau)\|^2\right) - \varepsilon_a\right)_+ \text{ in } (0,T), \tag{4}$$

$$u(0) = u_0, \dot{u}(0) = v_0, \beta(0) = \beta_0. \tag{5}$$

where the adhesion functional $j_{ad} : L^\infty(\Gamma_3) \times V \times V \to \mathbb{R}$ is given by

$$j_{ad}(\beta, u, v) = \int_{\Gamma_3} \left(p_\nu(u_\nu) - \gamma_\nu\beta^2 R_\nu(u_\nu)\right) v_\nu da + \int_{\Gamma_3} p_\tau(\beta)R_\tau(u_\tau).v_\tau da,$$

Let $\eta \in L^2(0,T;V')$ be given. We consider the following variational problem PV_η

Problem PV_η

$$(\ddot{u}_\eta, v)_{V'\times V} + (A(\varepsilon \dot{u}_\eta), \varepsilon(v))_{\mathcal{H}} + (\eta(t), v)_{V'\times V} = (f(t), v)_{V'\times V} \forall v \in V \text{a.e } t \in [0,T], \tag{6}$$

$$u_\eta(0) = u_0, \dot{u}_\eta(0) = v_0. \tag{7}$$

Lemma 1. *There exists a unique solution to problem PV_η satisfying the following regularity*

$$u_\eta \in W^{1,2}(0,T;V) \cap C^1(0,T;H), \ddot{u}_\eta \in L^2\left(0,T;V'\right).$$

Moreover, if u_i represents the solution of problem PV_η for $\eta = \eta_i \in L^2\left(0,T;V'\right), i = 1,2,$ then there exists $C > 0$ such that

$$\int_0^t \|\dot{u}_1(s) - \dot{u}_1(s)\|_V^2 ds \leq C \int_0^t \|\eta_1(s) - \eta_2(s)\|_{V'}^2 ds \quad \forall t \in [0,T].$$

We use the displacement field u_η obtained in **Lemma 1** and we consider the following initial value problem

Problem PV_η^β

Find the adhesion $\beta_\eta : [0,T] \to L^2(\Gamma_3)$ such that

$$\dot{\beta}_\eta = -\left(\beta_\eta\left(\gamma_\nu(R_\nu(u_{\eta\nu}))^2 + \gamma_\tau\|R_\tau(u_{\eta\tau})\|^2\right) - \varepsilon_a\right)_+ \text{ in } \Gamma_3 \times (0,T),$$

$$\beta(0) = \beta_0 \text{ in } \Gamma_3.$$

Lemma 2. *There exists a unique solution $\beta_\eta \in W^{1,\infty}(0,T;L^2(\Gamma_3)) \cap \mathcal{Z}$ to problem PV_η^β .*

Now, we introduce the operator $\Lambda : L^2\left(0,T;V'\right) \to L^2\left(0,T;V'\right)$ defined by

$$(\Lambda\eta(t), v)_{V'\times V} = \left(G\varepsilon\left(u_\eta(t)\right), \varepsilon(v)\right)_{\mathcal{H}} + j_{ad}(\beta_\eta, u_\eta, v).$$

Lemma 3. *The operator Λ has a unique fixed point $\eta^* \in L^2\left(0,T;V'\right)$.*

Now, we have all the ingredients to state and prove our principal theorem

Theorem 1. *Problem PV has a unique solution (u, σ, β) which satisfies*

$$u \in W^{1,2}(0,T;V) \cap C^1(0,T;H), \ddot{u} \in L^2\left(0,T;V'\right), \tag{8}$$

$$\sigma \in L^2\left(0,T;\mathcal{H}\right), Div\sigma \in L^2\left(0,T;V'\right), \tag{9}$$

$$\beta \in W^{1,\infty}(0,T;L^2(\Gamma_3)) \cap \mathcal{Z}. \tag{10}$$

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