Asymptotic behavior for nonlinear piezoelectric beams with magnetic effects

Hassan Messaoudi[1]

[1]Laboratory of Informatics and Mathematics, Department of Mathematics and Informatics, University of Souk-Ahras, P.O. Box 1553, Souk Ahras 41000, Algeria

hassanmessaoudi50conf@gmail.com

Piezoelectricity is the property that certain bodies have of being electrically polarized under the action of mechanical stress. In 1880, piezoelectricity was discovered by the brothers Pierre and Jacques Curie. These have shown that certain crystals (such as: quartz, lithium niobate, sugar cane, barium titanate and Rochelle salt) generate an electrical polarization under mechanical stress, that is to say produce an electrical charge under mechanical stress (the direct piezoelectric effect). The reversible phenomenon (the reverse piezoelectric effect), was theoretically stated by Gabriel Lippmann in 1881 and verified experimentally by the Curie brothers in the same year. Piezoelectric materials have been widely used in actuators or sensors because of their ability to convert electrical energy into mechanical energy and vice versa. These so-called intelligent materials can be used in various applications such as injection mechanisms, piezoelectric motors, sonars. In [5], Morris and Özer proposed a variational approach to construct a coupled model of piezoelectric beams with magnetic effect given by

$$\begin{cases} \rho v_{tt} - \alpha v_{xx} + \gamma\beta p_{xx} = 0, \text{ in } (0,L)\times(0,\infty), \\ \mu p_{tt} - \beta p_{xx} + \gamma\beta v_{xx} = 0, \text{ in } (0,L)\times(0,\infty), \end{cases} \tag{1}$$

where α, ρ, μ, γ, β and L are positive constants represent, respectively, elastic stiffness, the mass density, magnetic permeability, piezoelectric coefficient, water resistance coefficient of the beam and the length of the beam. Moreover, we consider the relationship

$$\alpha = \alpha_1 + \gamma^2\beta \text{ with } \alpha_1 > 0, \tag{2}$$

and the system (1) is equipped by the following boundary and initial conditions

$$\begin{cases} v(0,t) = p(0,t) = \alpha v_x(L,t) - \gamma\beta p_x(L,t) = 0, \\ \beta p_x(L,t) - \gamma\beta v_x(L,t) = -V(t)/h, \\ v(x,0) = v_0(x),\ v_t(x,0) = v_1(x),\ p(x,0) = p_0(x),\ p_t(x,0) = p_1(x), \end{cases} \tag{3}$$

where h, $V(t)$ represent, respectively, thickness of the beam, voltage applied at the electrode. Motivated by the above work, in this paper we consider the following problem

$$\begin{cases} \rho v_{tt} - \alpha v_{xx} + \gamma\beta p_{xx} + \chi(t) f(v_t) = 0, \text{ in } (0,L)\times(0,\infty), \\ \mu p_{tt} - \beta p_{xx} + \gamma\beta v_{xx} = 0, \text{ in } (0,L)\times(0,\infty), \\ v(0,t) = v_x(L,t) = p(0,t) = p_x(L,t) = 0,\ t\in(0,\infty), \\ v(x,0) = v_0(x),\ v_t(x,0) = v_1(x),\ p(x,0) = p_0(x),\ x\in(0,L), \\ p_t(x,0) = p_1(x),\ x\in(0,L), \end{cases} \tag{4}$$

where the functions v and p represent respectively, the longitudinal displacement of the center line, the total load of the electric displacement along the transverse direction at each point x. Other problems related to systems with nonlinear term [1-4].

The main result of this work is to study the asymptotic behavior of a piezoelectric beam system with a nonlinear damping term. First, by using the semi-group technique, we show the existence and uniqueness of the solution. Also, by using some properties of convex functions and Lyaponov functionals, we obtain general stability estimates.

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