The role of quarantine for containing the epidemic

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My work is concerned to study a degenerated susceptible, infected, quarantine, recovered (SIQR) epidemic model with spatial heterogeneity, and generalized nonlinear incidence functional. The problem associated to this work is:

$$\begin{cases} \dfrac{\partial S}{\partial t} = d_1 \Delta S + \Lambda(x) - f(x,S,I) - \mu(x)S, & t>0,\ x\in\Omega, \\ \dfrac{\partial I}{\partial t} = d_2 \Delta I + f(x,S,I) - (\mu(x)+\kappa_1(x)+\theta(x))I + \gamma(x)Q, & t>0,\ x\in\Omega, \\ \dfrac{\partial Q}{\partial t} = \theta(x)I - (\mu(x)+\kappa_2(x)+\gamma(x))Q, & t>0,\ x\in\Omega, \\ \dfrac{\partial R}{\partial t} = d_3 \Delta R + \kappa_1(x)I + \kappa_2(x)Q - \mu(x)Q, & t>0,\ x\in\Omega, \\ \dfrac{\partial S}{\partial \eta} = 0, \dfrac{\partial I}{\partial \eta} = 0, \dfrac{\partial R}{\partial \eta} = 0, & x\in\partial\Omega. \end{cases} \tag{1}$$

By the generalized Krein-Ruthman theorem we identify the basic reproduction number R_0, with it threshold role. For $R_0 \leq 1$, the disease will die out, which is guaranteed by the global asymptotic stability of the disease-free steady state, and for $R_0 > 1$, the disease will persist.

Lemma 1. (i) $\mathcal{R}_0 - 1$ *has the same sign as* $s(A)$*, where* $s(A)$ *is the spectral bound of* A*.(to obtain this we use Theorem 3.5, [1])*

(ii) *According to [2] we can prove that* $\mathcal{R}_0$ *is defined as*

$$\mathcal{R}_0 = \sup_{\phi\in H^1(\Omega),\phi\neq 0} \frac{\displaystyle\int_\Omega \frac{\partial f}{\partial I}(.,\hat{S},0)\phi^2 dx}{\displaystyle\int_\Omega \left(d_2|\nabla\phi|^2 + (\mu+\kappa_1+\theta\frac{\mu+\kappa_2}{\mu+\kappa_2+\gamma})\phi^2 \right) dx} \tag{2}$$

(iii) $\mathcal{R}_0 - 1$ *and* $s(A)$ *have the same sign as* η^0*, where* η^0 *is the principal eigenvalue of*

$$\begin{cases} d_2\Delta\phi - (\mu+\kappa_1+\theta\frac{\mu+\kappa_2}{\mu+\kappa_2+\gamma})\phi + \dfrac{\partial f}{\partial I}(.,\hat{S},0)\phi = \eta^0\phi & x\in\Omega, \\ \dfrac{\partial \phi}{\partial \eta} = 0 & x\in\partial\Omega, \end{cases} \tag{3}$$

Theorem 1. *Suppose that* $R_0 \leq 1$*. The disease free steady state is globally asymptotically stable*

Theorem 2. *Assume that* $\mathcal{R}_0 > 1$*, then for any initial data* $u_0 = (S_0, I_0, Q_0) \in \mathbb{X}^+ := C(\bar{\Omega}, \mathbb{R}^3_+)$ *with* $I_0 \not\equiv 0$*, or* $Q_0 \not\equiv 0$*, then there exists* $\delta > 0$ *such that the solution* $u = (S, I, Q)$ *of (1) satisfies*

$$\min\left\{ \liminf_{t\to\infty} S(.,t),\ \liminf_{t\to\infty} I(.,t),\ \liminf_{t\to\infty} Q(.,t) \right\} \geq \delta,\ \textit{uniformly for all } x\in\bar{\Omega}. \tag{4}$$

The main question that can be put in our study (due to the fact that the individuals in quarantine can repulse into the infected stage, and start infecting again): Is the quarantine effective in this case? To understand the role of quarantine better we compared our problem with SIR model. Numerical simulations are used to validate the theoretical finding.

1. H. R. Thieme, Spectral bound and reproduction number for infinite-dimensional population structure and time heterogeneity, SIAM J. Appl. Math. 70 (2009) 188-211.
2. W. Wang, X.-Q. Zhao, Basic reproduction number for reaction-diffusion epidemic models, SIAM J. Appl. Dyn. Syst., 11 (2012) 1652-1673.