Vaccination and treatment effects on the dynamic behavior of a TB model: application in Ukraine

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Many countries such as China, Europe, and the United States have low rates of TB mortality, while others like India struggle to control it. In this study, we analyze the dynamics of tuberculosis (TB) in Ukraine and examine vaccination effects and treatment effects. A mathematical model based on VSLIT is used for this purpose. In the first step, a qualitative study is conducted to understand the stability properties of disease-free and endemic equilibrium states. In the second step, using the least squares method and by utilizing the TB-reported data for Ukraine from 1990 to 2020, the model parameters are estimated. As a final step, numerical simulations are carried out to confirm the theoretical results.

In 2021, 1.6 million individuals worldwide died due to TB (including 187 000 people with HIV). TB is the second most common infectious killer in the world, behind COVID-19 (above HIV/AIDS), and the 13th largest cause of death overall. According to the World Health Organization (WHO).

Ukraine is one of nine nations worldwide with a high burden of multidrug-resistant TB and is regarded as a TB high priority country in the WHO European Region. TB is a bacterial infectious disease caused by bacillus Mycobacterium tuberculosis (MTB). Most epidemic diseases are modeled by a system of differential equations. Mathematical models have an important role in the planning of TB control programs. Bernoulli made the first work on mathematical modeling in 1766. Waaler et al, introduced the first mathematical model of TB in 1962, by dividing the whole population into three classes. And after that came many important works such as [1]-[2]. The main objective of the new Global WHO Strategy is to end TB in the world by 2035, and to achieve a zero rate of incidence, mortality, and suffering from this disease.

This study aims to analyze tuberculosis (TB) dynamics in Ukraine and investigate the vaccinations and treatment effects of the disease breaks. For this purpose, We proposed VSLIT mathematical model.

The population is divided into five classes: vaccinated V, susceptible S, latent(exposed) L, infected(TB active) I, and under treatment T. Hence, the total population is given as

$$N(t) = V(t) + S(t) + L(t) + I(t) + T(t).$$

The dynamics of TB infection are described by the following system of differential equations:

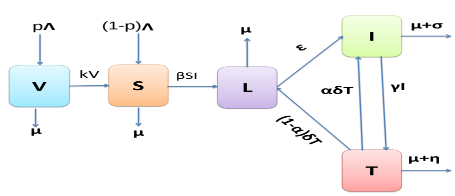

Figure 1: Flow chart of VSLIT model

$$\begin{cases} \dfrac{dV(t)}{dt} = p\Lambda - (k+\mu)V(t), \\ \dfrac{dS(t)}{dt} = (1-p)\Lambda + kV(t) - \beta S(t)I(t) - \mu S(t), \\ \dfrac{dL(t)}{dt} = \beta S(t)I(t) - (\epsilon+\mu)L(t) + (1-\alpha)\delta T(t), \\ \dfrac{dI(t)}{dt} = \epsilon L(t) + \alpha\delta T(t) - (\gamma+\mu+\sigma)I(t), \\ \dfrac{dT(t)}{dt} = \gamma I(t) - (\mu+\delta+\eta)T(t). \end{cases} \tag{1}$$

Equipped with no-negative initial conditions.

The model parameters are estimated by minimizing the error between actual TB incidence data from 1990 to 2020 in Ukraine [3], and the solution of the proposed model (1). The objective function is given by

$$\psi = \sum_{i=1}^{n} (I_{t_i} - I^*_{t_i})^2,$$

where $I^*_{t_i}$ denotes the actual TB infected case and I_{t_i} are the corresponding model solution at time t_i, n is the number of available actual data. To minimize the objective function we have used the Matlab function 'fitnlm' which solves nonlinear regression problems by the Levenberg-Marquardt algorithm.

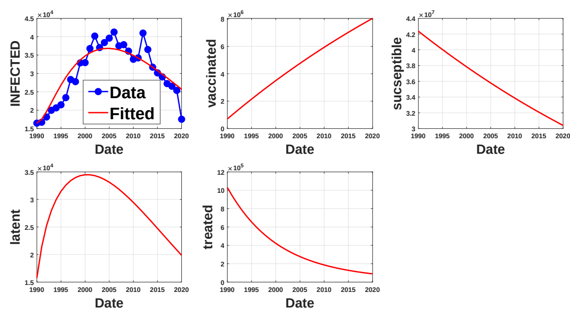

Figure 2: Data fitting of the number of TB cases in Ukraine

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