Nonparametric estimation of the reliability characteristics of a standby system

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It is well known that the reliability of engineering systems can be enhanced by increasing the level of redundancy used. Integrating redundancy into systems is particularly effective when random failures predominate or when the reliability is crucial for the safety of people and the environment, such as aeronautics or nuclear... In this work, we consider the standby system with reparation, consisting initially of an active component and C-1 inactive (standby) components that become active in the event of a breakdown of the operating component. First, we showed that, under certain hypotheses, this system can be faithfully modeled by the $GI/D/1/C$ queue. Secondly, we illustrated how to use the nonparametric "discrete-kernel" method to estimate the reliability characteristics of the considered standby system, when we retain the $GI/D/1/C$ queue as its mathematical model, for more details on the approach followed see [1] and [2]. Besides, to support and illustrate our proposals, comparative numerical applications, based on simulated samples, have been carried out.

Modeling of a standby system by the $GI/D/1/C$ queue

The aim of the present section it is to concretize the various concepts related to the $GI/D/1/C$ queues and their model. Let consider a standby system consisting, initially, of an active component and $C-1$ inactive (standby) components that become active in the event of a breakdown of the operating component. In addition, this system works under the following assumptions:

- All components are identical, at least in the sense of their reliability characteristics.
- In the event of a breakdown of an active component, a repairer will proceed immediately to replace it by an equivalent component in standby redundancy.
- The system is failed on date t if the C components are all failed on this date.
- The repairer can only replace one failed component at a time.
- The failed components are replaced in chronological order of their failures.
- The lifetime of an operating component is a random variable X whose distribution is an arbitrary (general) law $A(t)$ of average $1/\lambda$ and the standby components are assumed not to fail while in the idle state.
- The replacement times R of the failed components are practically equal ($Var(R) \approx 0$), of average time $1/\mu$ ($E(R) = 1/\mu$).
- The system is equipped with a failure-free switching detector that ensures instantaneous switching from the failed component to the standby component (perfect switching).

Under the above assumptions and that there are a sufficient number of replacement components in the stock, then this system can be faithfully modeled by the $GI/D/1/C$ queue. The duality between the starting parameters of the system and those of the $GI/D/1/C$ queue is summarized in Table (1).

Standby System parameters	$GI/D/1/C$ queue parameters
• A failed component.	• The customer.
• The repairer (one repairer).	• The server (one server).
• Replacement of a component.	• The service.
• C: The total number of components in the system.	• C: The system capacity.
• λ: The failure rate ($1/\lambda$: The mean lifetime of a component.).	• λ: The arrival rate ($1/\lambda$: The mean time between two consecutive arrivals.).
• $A(t)$: The lifetime distribution of a component.	• $A(t)$: The distribution of the inter-arrival time.
• Replacement time.	• Service time.
• Replacement time is a constant ($1/\mu$).	• The distribution of the service time is a deterministic distribution ($S(t) = \mathbf{1}_{\{t=\mu^{-1}\}}$).
• The failed components are replaced in chronological order of their failures.	• The queue discipline is *First In, First Out* (FIFO).

Table 1: Equivalence between the starting parameters of the standby system and those of the $GI/D/1/C$ queue.

Kernel density estimation

Obviously, the density f generates the sample, but the question that arises is that when we have a sample, can we approximatively recreate (estimate) its density?

To estimate the density f, a first so-called parametric approach consists of assuming that f belongs to a family of densities which can be described by a certain number of real parameters ($f(x, \theta)/\theta \in \mathbb{R}^d$ with $d \in \mathbb{N}^*$). The statistician who chooses such an approach has a good a priori knowledge of the random phenomenon. However, when no information is available on the phenomenon studied or the parameter is of a very large dimension (d is large enough), the application of a parametric model is not satisfactory. To overcome the insufficiencies and the defects of the parametric families, a second so-called nonparametric approach consists of estimating this unknown density function f without specifying, beforehand, its form.

Let X_1, ..., X_n be independent and identically distributed ($i.i.d.$) random variables with an unknown probability density function f on $\mathfrak{R} \subseteq \mathbb{R}$. The kernel estimator $\hat{f}$ of the density f is defined as follow:

$$\hat{f}(x) = \frac{1}{n} \sum_{i=1}^{n} K_{x,h}(X_i), \qquad x \in \mathfrak{R}. \tag{1}$$

where $h = h(n) > 0$ is an arbitrary sequence of smoothing parameters that fulfills $\lim_{n\to\infty} h(n) = 0$ and $K_{x,h}$ is the kernel function, which is typically a probability density function with finite variance, of target x and the smoothing parameter h on the support $\aleph_{x,h} = \aleph_x$ (does not depend on h).

Numerical application The purpose of this section is to analyze numerically the impact of the smoothing parameter and the kernel choosing, on the performances of the reliability characteristics estimators of the standby system when this latest is modeled by an $GI/D/1/C$ queue.

1. Afroun F., Aissani D. Hamadouche D. Nonparametric approximation of the characteristics of the $D/G/1$ queue with finite capacity. Int. J. Computing Science and Mathematics, 2022, Vol.16, No.2, 170–180 p.
2. Cherfaoui M., Boualem M., Aissani D. and Adjabi S. Choix du paramиtre de lissage dans l'estimation a noyau d'une matrice de transition d'un processus semi-markovien. C. R. Acad. Sci. Paris, Ser. I, 2015, Vol.353, No.3, 273–277 p.