On the algebra of derivations of the Leibniz algebra $\mathbf{Lei}_3(3, F)$

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In this abstract we describe the algebra of derivation of some nilpotent Leibniz algebra, having dimension 3.

Let L be an algebra over a field F with the binary operations $+$ and $[\cdot,\cdot]$. Then L is called a *Leibniz algebra* (more precisely a *left Leibniz algebra*), if it satisfies the (left) Leibniz identity

$$[[a,[b,c]] = [[a,b],c] + [b,[a,c]] \text{ for all } a,b,c \in L.$$

Leibniz algebras appeared first in the paper of A. M. Bloh [1], but the term “Leibniz algebra” appears in the book of J.-L. Loday [2].

Denote by $\mathbf{End}_{[,]}(L)$ the set of all linear transformations of L, then L is an associative algebra by the operation $+$ and $\circ$. As usual, $\mathbf{End}_{[,]}(L)$ is a Lie algebra by the operations $+$ and $[,]$, where $[f,g] = f \circ g - g \circ f$ for all $f, g \in \mathbf{End}_{[,]}(L)$.

A linear transformation f of a Leibniz algebra L is called a *derivation*, if

$$f([a,b]) = [f(a),b] + [a,f(b)] \text{ for all } a,b \in L.$$

Let $\mathbf{Der}_{[,]}(L)$ be the subset of all derivations of L. It is possible to prove that $\mathbf{Der}_{[,]}(L)$ is a subalgebra of a Lie algebra $\mathbf{End}_{[,]}(L)$. $\mathbf{Der}_{[,]}(L)$ is called the *algebra of derivations* of a Leibniz algebra L.

Theorem 1. *Let $\mathbf{D}$ be an algebra of derivations of the Leibniz algebra $\mathbf{Lei}_3(3,F)$. Then the following assertions hold:*

(i) *$\mathbf{D}$ is a semidirect sum of an ideal $\mathbf{A}$ and a subalgebra of dimension 1, generated by derivation f_1 such that $f_1(a_1) = a_1, f_1(a_2) = a_2, f_1(a_3) = 2a_3$;*

(ii) *$\mathbf{A}$ is a semidirect sum of an ideal $\mathbf{N}$ of $\mathbf{D}$ and a subalgebra of dimension 1, generated by derivation f_2 such that $f_2(a_1) = a_2$, $f_2(a_2) = a_2$, $f_2(a_3) = a_3$;*

(iii) *an ideal $\mathbf{N}$ is abelian, $\mathbf{N} = Ff_3 \oplus Ff_4$, where*

$$f_3(a_1) = a_3, f_3(a_2) = 0, f_3(a_3) = 0, f_4(a_1) = 0, f_4(a_2) = a_3, f_4(a_3) = 0.$$

Moreover, $[f_1, f_4] = f_4$, $[f_1, f_3] = f_3$, $[f_1, f_2] = 0$, $[f_2, f_4] = -f_3$, $[f_2, f_3] = f_3$;

(iv) *an algebra $\mathbf{D}$ is isomorphic to a Lie subalgebra of matrices, having the following form*

$$\begin{pmatrix} \alpha_1 & 0 & 0 \\ \alpha_2 & \alpha_1 + \alpha_2 & 0 \\ \alpha_3 & \beta_3 & 2\alpha_1 + \alpha_2 \end{pmatrix}, \text{ where } \alpha_1, \alpha_2, \alpha_3, \beta_3 \in F.$$

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1. Bloh A. M. On a generalization of the concept of Lie algebra. Doklady AN USSR, 1965, 165, 471 – 473 (in Russian).
2. Loday J.-L. Une version non commutative des algebres de Lie; les algebres de Leibniz. Enseign. Math., 1993, 39, 269 – 293.