

# PRE-TORSION THEORIES ON CONCRETE CATEGORIES

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*Pre-torsion theories* have been introduced in [1] in order to generalize the notion of torsion theory (and hence the classical categorical homology theory) to non-pointed categories. To this regard, it suffices to choose a suitable subclass  $\mathfrak{Z}$  of *trivial objects* and, next, to generalize the classical notions of kernels and cokernels to those of  $\mathfrak{Z}$ -prekernels and  $\mathfrak{Z}$ -precokernel, respectively.

In the present talk, based on the results obtained in [2], we introduce the notion of *proper Moore-concrete subcategory* of a given concrete category  $\mathfrak{C}$  over a base category  $\mathfrak{X}$  and show that it agrees with that of reflective modification of  $\mathfrak{C}$ . This allows us to associate with any object  $C$  of  $\mathfrak{C}$  an object in  $\mathfrak{B}$  that we call the  $\mathfrak{B}$ -closure of  $C$ , and with any morphism in  $\mathfrak{C}$  a unique morphism between the corresponding  $\mathfrak{B}$ -closures that we call the  $\mathfrak{B}$ -closure extension.

Next, we analyze some different conditions to be put on short sequences of morphisms or on short pre-exact sequences in  $\mathfrak{C}$  in order to get informations on the corresponding  $\mathfrak{B}$ -closure extensions and on the  $\mathfrak{B}$ -closure of the various terms of the associated short sequences of  $\mathfrak{B}$ -closure extensions in  $\mathfrak{B}$ . Furthermore, we induce pretorsion theories on proper Moore-concrete subcategories starting from pretorsion theories on the ambient category  $\mathfrak{C}$  and, more in general, we study the interrelations between pretorsion theories on  $\mathfrak{C}$  and proper Moore-concrete subcategories.

Finally, we generalize the results holding for the category of pre-orders and monotone functions given in [1], exhibiting some interesting characterizations of trivial morphisms and of prekernels of a proper Moore-concrete subcategory  $\mathfrak{B}$  of a functor-structured category when we choose  $\mathfrak{Z}$  as the class of all projective objects, and constructing a quotient pointed category  $\mathfrak{B}/\mathfrak{R}^*$  inducing a correspondence between  $\mathfrak{Z}$ -prekernels [resp.  $\mathfrak{Z}$ -precokernels] in  $\mathfrak{B}$  and kernels [resp. weak cokernels] in  $(\mathfrak{B}/\mathfrak{R})$ .

1. Facchini A., Finocchiaro C. A., Pretorsion theories, stable category and preordered sets. *Annali di Matematica Pura ed Applicata*, 2020, 199, 1073–1089.
2. Infusino F. G., Prekernels of Topologically Axiomatized Subcategories of Concrete Categories, *Journal of Algebra*, 2023, 622, 469–505.