On Closedness of some permutative posemigroup identities

Rizwan Alam
Aligarh Muslim University, Aligarh, India
rizwanamuba@gmail.com

As we know that all non-trivial permutation identities are not preserved under epimorphisms of partially ordered semigroups. In this paper towards this open problem, first we show that certain non-trivial identities in conjunction with the permutation identity $z_1z_2\cdots z_n = z_{i_1}z_{i_2}\cdots z_{i_n}$ $(n \geq 2)$ with $i_n \neq n$ $[i_1 \neq 1]$ are preserved under epimorphisms of partially ordered semigroups. Further, we extend a result of Ahanger and Shah which showed that the center of a partially ordered semigroup S is closed in S and show that the normalizer of any element of a partially ordered semigroup S is closed in S.

Theorem 1. *All non trivial identities of the form* $z_1^{p_1}z_2^{p_2}\cdots z_r^{p_r} = z_1'^{q_1}z_2'^{q_2}\cdots z_{r'}'^{q_{r'}}$, *where* $p_1, p_2, \ldots, p_r, q_1, q_2, \ldots, q_{r'} > 0$, *are preserved under epis of posemigroups in conjunction with the permutation identity (2.1) with* $i_n \neq n$ $[i_1 \neq 1]$.

Definition 1. A semigroup S is said to be permutative if S satisfies a permutation identity

$$z_1z_2\cdots z_n = z_{i_1}z_{i_2}\cdots z_{i_n}, \tag{1}$$

where i is a non trivial permutation of the set $\{1, 2, \ldots, n\}$ and $i_1, i_2, \ldots, i_n$ are the images of $1, 2, \ldots, n$ under the permutation i respectively. A posemigroup S is said to be a permutative if it is so as a semigroup.

Theorem 2. *Following type of non trivial identities are preserved under epis of posemigroups in conjunction with any permutation identity* (1) *with* $i_n \neq n$ $[i_1 \neq 1]$

Theorem 3. *Let* S *be any posemigroup and* $a \in S$. *Then* $N(a)$ *is closed in* S.