Mappings with finite length distortion on Riemann surfaces

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The class of mappings with finite length distortion was introduced in [1] for $\mathbb{R}^n$, $n \geqslant 2$, see also [2]. It was shown in [3] that such mappings, generally speaking, are not mappings with finite distortion by Iwaniec investigated on Riemann surfaces in [4]–[5]. This class is a natural generalization of the classes of isometries and quasi–isometries. We prove criteria in terms of dilatations K_f for the continuous and homeomorphic extension to the boundary of these mappings f between domains in **compactifications by Kerekjarto-Stoilow** of Riemann surfaces, see definitions and notations in [4]–[6]. For instance:

Theorem 1. *Let $\mathbb{S}$ and $\mathbb{S}^*$ be Riemann surfaces, D and D^* be domains in $\overline{\mathbb{S}}$ and $\overline{\mathbb{S}^*}$, correspondingly, $\partial D \subset \mathbb{S}$ and $\partial D^* \subset \mathbb{S}^*$, D be locally connected on its boundary and let ∂D^* be weakly flat. Suppose that $f : D \to D^*$ is a homeomorphism of finite length distortion with $K_f \in L^1_{\rm loc}$. Then the mapping $g = f^{-1} : D^* \to D$ is extended by continuity to a mapping $\widetilde{g} : \overline{D^*} \to \overline{D}$ and $\widetilde{g}(\partial D^*) = \partial D$.*

Theorem 2. *Let $\mathbb{S}$, $\mathbb{S}^*$ be Riemann surfaces, D, D^* be domains on $\overline{\mathbb{S}}$, $\overline{\mathbb{S}^*}$, $\partial D \subset \mathbb{S}$, $\partial D^* \subset \mathbb{S}^*$, D be locally connected on ∂D, ∂D^* be strongly accessible. Suppose that $f : D \to D^*$ is a homeomorphism with finite length distortion and, for all $p_0 \in \partial D$,*

$$\int\limits_0^{\varepsilon(p_0)} \frac{dr}{||K_f||\,(p_0,r)} \;=\; \infty\ , \qquad ||K_f||\,(p_0,r) \;:=\; \int\limits_{h(p,p_0)=r} K_f(p)\; ds_h(p)\ . \quad (1)$$

Then the mapping f is extended by continuity to a mapping $\widetilde{f} : \overline{D} \to \overline{D^*}$ and $\widetilde{f}(\partial D) = \partial D^*$.

For proofs and new criteria, see the extended version (in English) expected in Dopovidi of NASU.

References

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