On some finite difference properties of conformal mappings

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Let two simply connected domains G_1 and G_2 be given. Let domains G_1 and G_2 be bounded by the smooth Jordan curves Γ_1 and Γ_2. Let $\tau_1 = \tau_1(s_1)$ be the angle between the tangent to the curve Γ_1 and the positive real axis considered as the function of the arc length $s_1 = s_1(z)$ on the curve Γ_1. Let $\tau_2 = \tau_2(s_2)$ be the angle between the tangent to the curve Γ_2 and the positive real axis considered as the function of the arc length $s_2 = s_2(w)$ on the curve Γ_2.

Let consider the function $w = f(z)$ realizing homeomorphism between the closure $\overline{G_1}$ of the domain G_1 onto closure $\overline{G_2}$ of the domain G_2 conformal in open domain G_1.

The relationship between properties of the function describing the boundaries of the considered domains and properties of the considered homeomorphism is investigated. The results are obtained as estimates for the general uniform and the non-centralized local arithmetic moduli of smoothness of arbitrary order.

In partial case when moduli of smoothness of arbitrary order k for the functions characterizing boundaries of the domains satisfy conditions

$$\omega_k(\tau_1(s_1), \delta) = O(\delta^\alpha) \quad (\delta \to 0),$$
$$\omega_k(\tau_2(s_2), \delta) = O(\delta^\alpha) \quad (\delta \to 0), \quad 0 < \alpha < k,$$

then modulus of smoothness of the same order k for the derivative of the functions realizing conformal mapping satisfies condition

$$\omega_k(f'(\xi), \delta) = O(\delta^\alpha) \quad (\delta \to 0)$$

with the same index α.

Certain estimates were received for moduli of smoothness of arbitrary order for the higher order derivatives of considered functions.