Boundary value problems for singular systems of difference equations

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We investigate the problem of finding bounded solutions $z(k) \in \mathbb{R}^n, \quad k \in \Omega$ of linear Noetherian $(n \neq \upsilon)$ boundary value problem for a system of linear difference-algebraic equations [1, 2]

$$A(k)z(k+1) = B(k)z(k) + f(k), \quad \ell z(\cdot) = \alpha, \quad \alpha \in \mathbb{R}^{\upsilon}; \tag{1}$$

here $A(k),\ B(k) \in \mathbb{R}^{m \times n}$ are real matrices and $f(k)$ are real column vectors, $\ell z(\cdot) : \mathbb{R}^n \to \mathbb{R}^{\upsilon}$ is a linear bounded vector functional defined on a space of bounded functions. The problem of finding bounded solutions $z(k)$ of a boundary value problem for a linear non-degenerate $(\det B(k) \neq 0,\ k \in \Omega)$ system of first-order difference equations $z(k+1) = B(k)z(k) + f(k), \quad \ell z(\cdot) = \alpha \in \mathbb{R}^{\upsilon}$ was solved by A.A. Boichuk [1]. We investigate the problem of finding bounded solutions linear Noetherian boundary value problem for a system of linear difference-algebraic equations (1) in case [3]

$$A(k) = R_0(k) \cdot J_{\sigma_0} \cdot S_0(k), \quad J_{\sigma_0} := \begin{pmatrix} I_{\sigma_0} & O \\ O & O \end{pmatrix}, \quad 1 \leq \text{rank } A(k) = \sigma_0, \quad k \in \Omega;$$

here, $R_0(k)$ and $S_0(k)$ are nonsingular matrices. The nonsingular change of the variable

$$y(k+1) = S_0(k)z(k+1)$$

reduces system (1) to the form [3]

$$A_1(k)\varphi(k+1) = B_1(k)\varphi(k) + f_1(k); \tag{2}$$

Under the condition [3], when matrixes $A_1^+(k)B_1(k)$ and column vectors $A_1^+(k)f_1(k)$, are bounded and also

$$P_{A^*}(k) \neq 0, \quad P_{A_1^*}(k) \equiv 0, \tag{3}$$

we arrive at the problem of construction of solutions of the linear difference-algebraic system

$$\varphi(k+1) = A_1^+(k)B_1(k)\varphi(k) + \mathfrak{F}_1(k, \nu_1(k)), \quad \nu_1(k) \in \mathbb{R}^{\rho_1}; \tag{4}$$

here,

$$\mathfrak{F}_1(k, \nu_1(k)) := A_1^+(k)f_1(k) + P_{A_{\varrho_1}}(k)\nu_1(k),$$

$A_1^+(k)$ is a pseudoinverse (by Moore – Penrose) matrix. In addition, $P_{A_1^*(k)}$ is a matrix-orthoprojector: $P_{A_1^*}(k) : \mathbb{R}^{\sigma_0} \to \mathbb{N}(A_1^*(k))$, $P_{A_{\rho_1}}(k)$ is an $(\rho_0 \times \rho_1)$-matrix composed of ρ_1 linearly independent columns of the $(\rho_0 \times \rho_0)$-matrix-orthoprojector: $P_{A_1}(k) : \mathbb{R}^{\rho_0} \to \mathbb{N}(A_1(k))$. Thus, the following lemma is proved.

Lemma 1. *For the first-order degeneration difference-algebraic system (1) has a solution of the form*

$$z(k, c_{\rho_0}) = X_1(k)\, c_{\rho_0} + K[f(j), \nu_1(j)](k), \quad c_{\rho_0} \in \mathbb{R}^{\rho_0};$$

which depends on the arbitrary continuous vector-function $\nu_1(k) \in \mathbb{R}^{\rho_1}$, where $X_1(k)$ is fundamental matrix, $K[f(j), \nu_1(j)](k)$ is the generalized Green operator of the Cauchy problem for the linear difference-algebraic system (1).

References

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