On notions of Q-independence and Q-identical distributiveness

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In article [1] A.M. Kagan and G.J. Székely introduced a notion of Q-independent and Q-identical distributed random variables.

Definition 1. Let $(X_1, \dots, X_n)$ be a random vector with the characteristic function $\varphi(t_1, \dots, t_n)$. Denote by $\varphi_1(t), \dots, \varphi_n(t)$ the characteristic functions of the random variables $X_1, \dots, X_n$ respectively. The random variables $X_1, \dots, X_n$ are said to be *Q-independent* if for all $t_1, \dots, t_n \in \mathbb{R}$ the condition

$$\varphi(t_1, \dots, t_n) = \varphi_1(t_1) \cdots \varphi_n(t_n) \exp(q(t_1, \dots, t_n)),$$

where $q(t_1, \dots, t_n)$ is a polynomial such that $q(0, \dots, 0) = 0$, holds.

Definition 2. Random variables X and Y with the characteristic functions $\varphi_X(t)$ and $\varphi_Y(t)$ respectively are said to be *Q-identically distributed* if for all $t \in \mathbb{R}$ the condition

$$\varphi_X(t) = \varphi_Y(t) \exp(q(t)),$$

where $q(t)$ is a polynomial such that $q(0) = 0$, holds.

A.M. Kagan and G.J. Székely describe in [1] a wide class of polynomials for Definition 2. They also give an important example for Definition 1 with polynomial $q(t_1, t_2) = t_1 t_2$. We give a complete description of polynomials which appear in these definitions. The main tool of our investigation is paper [2] of A.A. Goldberg.

References

[1] A.M. Kagan, G.J. Székely. An analytic generalization of independence and identical distributiveness. *Statistics and Probability Letters*, 110 : 244-248, 2016.

[2] A.A. Goldberg. On a question by Yu.V. Linnik. *Doklady AN SSSR*, 211 : 31-34, 1973. (in Russian)