

Point similarity transformations in the study of differential equations

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There is no general theory of integration for nonlinear partial differential equations (PDEs), so the search for integrable cases and exact solutions often relies on the application of appropriate changes of variables. If two PDEs are related by a nondegenerate point transformation then these equations are called similar and the respective transformation is called the *similarity transformation* [1].

The study of various problems for classes of PDEs including group classification problems relies on the use of such transformations within a given class, i.e. transformations from the equivalence groupoid of the class. In this talk we briefly recall the related notions of admissible transformations, various types of equivalence groups, and the normalization property of differential equations [2, 3]. Using illustrative examples, we show the application of equivalence groupoids in solving group classification problems [4] and studying integrability [5].

Then we discuss the method of mappings between classes of PDEs suggested in [6]. This method uses transformations that do not preserve the form of equations within the class in contrast to equivalence transformations but map the initial class to another one that is simpler for investigation. This method has proven to be a powerful tool for solving group classification problems.

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