On topology of spaces of persistence diagrams

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The Topological Data Analysis (TDA) provides metric and topological tools for investigation of large data arrays. Usually, the sets of data possess a structure of filtered topological space and can be investigated be means of persistent homology. In turn, the persistence homology classes can be represented by the persistence diagrams indication the "birth" and "death" of every such class (see, e.g., [1]–[3]).

The persistence diagrams are therefore important objects in the TDA. The set of persistence diagrams can be endowed with various metrics, in particular, the bottleneck metric. We also endow the set of persistence diagrams with a nonmetrizable topology such that the obtained topological space is homeomorphic to the countable direct limit of infinite tower of Euclidean spaces $\mathbb{R}^\infty$. Also, completions of the metric spaces of persistence diagrams are considered.

The (completed) spaces of persistence diagrams are infinite-dimensional. The aim of the talk is to apply methods of infinite-dimensional topology (in particular, characterization theorems for infinite-dimensional manifolds) to description of topology of these spaces. In particular, we prove that some of these spaces are homeomorphic to the pre-Hilbert spaces of finite sequences. Also, we show that the complement of the space of persistence diagrams is locally homotopy negligible in its completion. This naturally leads to the problem whether the completed space of persistence diagrams is an absolute retract.

References

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