On pseudo-harmonic functions defined on $k-$connected domain

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The problem of topological classification is central in topology. By V.Sharko, S. Maksymenko and E. Polulyakh, the conditions of topological conjugacy of many types of functions were successfully proved, for example, see [1, 3, 4].

Let $D \subset \mathbb{R}^2$ be an oriented finitely connected domain whose boundary consists of closed Jordan curves $\gamma_0, \gamma_1, \dots, \gamma_k$ and $f : D \to \mathbb{R}$ be a pseudo-harmonic function. It is known it satisfies the following conditions:

A) $f\mid_{\gamma_i}$ is a continuous function with a finitely many local extrema for $i = \overline{0, k}$;

B) $f\mid_{\mathrm{Int}D}$ has a finitely many critical points and each of them is a saddle point (in the neighborhood of it f has a representation like $Rez^n + const$, $z = x + iy$ and $n \geq 2$), where $\mathrm{Int}D = D \backslash_{(\bigcup\limits_i \gamma_i)}$.

In [2] authors researched a case of $k = 0$: for such functions, a topological invariant is constructed, its main properties, the criterion of their topological equivalence and conditions of realization of some type of graphs as given invariant are proved.

We will be interested in a case $k \geq 1$. A combinatorial invariant $\Delta(f)$ of f was constructed by the author in [5]. It is a mixed pseudograph (graph with multiple edges and loops) with a strict partial order on vertices. In [5, 6] the author proved some properties of $\Delta(f)$.

Definition 1. $C-$cycle of $\Delta(f)$ is a simple cycle whose arbitary pair of adjacent vertices v_i and v_{i+1} is comparable.

Theorem 2. *Let $\Delta(f)$ be a combinatorial invariant of pseudo-harmonic function f defined on $k-$connected domain. There are $(k + 1)$ $C-$cycles at $\Delta(f)$.*

By $\mathcal{L}(f)$ we denote a set of level curves of critical and semiregular values that contain critical and boundary critical points. Let consider $D_j, j = \overline{1, N}$, $N \in \mathbb{N}$, which is connected component of $\overline{D \setminus \mathcal{L}(f)}$.

A closed domain D_j has $R-$type if $D_j \bigcap \partial D = \emptyset$. A closed domain D_j has $St-$type ($Se-$type) if $D_j \bigcap \partial D \neq \emptyset$ and $D_j \bigcap \partial D = \{\alpha, \beta\}$ ($D_j \bigcap \partial D = \{\alpha\}$), where $\alpha \bigcap \beta = \emptyset$ and $\alpha, \beta \subset \partial D$ ($\alpha \subset \partial D$).

Theorem 3. *For any index j a closed domain D_j has one of the following types: R, St or Se.*

Theorem 4. *Two pseudo-harmonic functions f and g defined on $k-$connected domain are topologically equivalent iff there exists an isomorphism $\phi : \Delta(f) \to \Delta(g)$ which preserves their strict partial orders and the orientations.*

References

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