Symmetric analytic functions on some Banach spaces

Taras Vasylyshyn
(Vasyl Stefanyk Precarpathian National University, 57 Shevchenka Str., Ivano-Frankivsk 76018, Ukraine)
E-mail: `taras.v.vasylyshyn@gmail.com`

Let X be a Banach space, which has a symmetric structure, like has a symmetric basis or is rearrangement invariant. It is natural to consider polynomials and analytic functions on X, which are invariant (symmetric) with respect to a group of operators $G(X)$ acting on X, which preserve this structure. In particular, if X is a rearrangement invariant Banach space of Lebesgue measurable functions on some Lebesgue measurable set $\Omega \subset \mathbb{R}$ of nonzero measure, then $G(X)$ used to be the group of operators $B_\sigma : X \ni x \mapsto x \circ \sigma \in X$, where σ is a bijection of Ω, which preserves the measure. In some cases, algebras of continuous symmetric polynomials on such spaces have algebraic bases (see definition below), which gives us the opportunity to describe spectra of algebras of symmetric analytic functions on these spaces.

Definition 1. A mapping $f : X \to \mathbb{C}$ is called an *algebraic combination* of mappings $f_1, \ldots, f_k : X \to \mathbb{C}$ if there exists a polynomial $Q : \mathbb{C}^k \to \mathbb{C}$ such that $f(x) = Q(f_1(x), \ldots, f_k(x))$ for every $x \in X$.

Definition 2. A set of mappings $\mathcal{B}$ is called an *algebraic basis* of some algebra of mappings $\mathcal{A}$, if every element of $\mathcal{A}$ can be uniquely represented as an algebraic combination of some elements of $\mathcal{B}$.

Symmetric polynomials and symmetric analytic functions on some non-separable Banach spaces were studied in [1, 2]. In particular, in [1] it was constructed an algebraic basis of the algebra of continuous symmetric polynomials on the complex Banach space L_∞ of all Lebesgue measurable essentially bounded complex-valued functions on $[0, 1]$. Also, in [1] the spectrum (the set of all continuous linear multiplicative functionals) of the Fréchet algebra $H_{bs}(L_\infty)$ of all entire symmetric functions of bounded type on L_∞ was described. In [5] it was shown that the Fréchet algebra $H_{bs}(L_\infty)$ is isomorphic to the Fréchet algebra of all entire functions on its spectrum. In [2] it was shown that the trivial polynomial is the unique continuous symmetric polynomial on the complex Banach space of all Lebesgue measurable essentially bounded complex-valued functions on $[0, +\infty)$.

Symmetric polynomials on Cartesian powers of some Banach spaces were studied in [3, 4, 6]. In particular, in [4] and [3] there were constructed algebraic bases of algebras of continuous symmetric polynomials on Cartesian powers of complex Banach spaces of Lebesgue measurable integrable in a power p, where $1 \le p < +\infty$, complex-valued functions on $[0, 1]$ and $[0, +\infty)$ respectively. In [6] it was constructed an algebraic basis of the algebra of continuous symmetric polynomials on the Cartesian power of L_∞.

References

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