Extension of continuous operators on $C_b(X, E)$ with the strict topology

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In the paper [2] Nowak has developed the theory of continuous linear operators on the space $C_b(X, E)$ of bounded continuous functions $f : X \to E$, where X is a completely regular Hausdorff space and E is a Banach space. Then the space $C_b(X, E)$ is equipped with the strict topology β. Recall that β is generated by the family of the seminorms:

$$p_v(f) := \sup_{t \in X} |v(t)| \, \|f(t)\|_E \quad \text{for} \quad f \in C_b(X, E),$$

where $v : X \to \mathbb{R}$ is a bounded function such that for every $\varepsilon > 0$, $\{t \in X : |v(t)| \geq \varepsilon\}$ is a compact subset of X. For X being a locally compact space β coincides with the original strict topology that was introduced in 1958 by Buck [1]. The Riesz Representation Theorem for continuous linear operators $T : C_b(X, E) \to F$ was obtained, where F is a Banach space (see [2, Theorem 3.1]).

Let $\mathcal{L}^\infty(\mathcal{B}o, E)$ stand for the set of all bounded strongly $\mathcal{B}o$-measurable functions $g : X \to E$. Then $\mathcal{L}^\infty(\mathcal{B}o, E)$ can be equipped with the natural mixed topology $\gamma_{\mathcal{L}^\infty(\mathcal{B}o,E)}$. Note that if X is separable (resp. E is separable), then

$$C_b(X, E) \subset \mathcal{L}^\infty(\mathcal{B}o, E).$$

The aim of this talk is to present some results concerning the problem of extension of different classes of $(\beta, \| \cdot \|_F)$-continuous linear operators $T : C_b(X, E) \to F$ to the corresponding classes of $(\gamma_{\mathcal{L}^\infty(\mathcal{B}o,E)}, \| \cdot \|_F)$-continuous linear operators $\overline{T} : \mathcal{L}^\infty(\mathcal{B}o, E) \to F$.

References

[1] Robert C Buck. Bounded continuous functions on a locally compact space, *Michigan Math. J.*, 5: 95–104, 1958.
[2] Marian Nowak. A Riesz representation theory for completely regular Hausdorff spaces and its applications, *Open Math.*, 14: 474–496, 2016.