Topology of the set of factorable almost periodic matrix functions

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A classical Wiener-Hopf factorization of (matrix) functions defined on the unit circle $\mathbb{T}$ or the real line $\mathbb{R}$ is their representation as a product of three factors, the left/right analytic and invertible inside/outside $\mathbb{T}$ (respectively, in the upper/lower half-pane), and the diagonal middle factor with the diagonal entries of some special form.

For several classes of functions, scalar or matrix valued, the factorablity is equivalent to invertibility. These classes include the Wiener algebra W of functions with absolutely convergent Fourier series, continuous (matrix) functions, etc. The same is true for the (scalar) almost periodic functions with absolutely convergent Bohr-Fourier series (the algebra APW), or the algebra AP of all Bohr almost periodic functions.

This property is lost, however, in transition to **matrix**-valued AP or APW functions. Moreover, the respective factorability criteria are presently not know even in the case of 2-by-2 triangular matrix-functions in these classes. The configuration of the set GLF of all factorable AP or APW matrix functions within the respective group GL of invertible matrix-functions is therefore a non-trivial issue.

As was established jointly with A. Brudnyi and L. Rodman, there are infinitely many pathwise connected components of GL not intersecting with GLF, and even with the closed subgroup of GL generated by it.