About multiary webs

F. M. Sokhatsky
(Vasyl' Stus Donetsk National University, Vinnytsia, Ukraine)
E-mail: `fmsokha@ukr.net`

In [1] binary k-webs are investigated. In [2] algebraic aspects of web geometry, namely its connections with the quasigroup and loop theory, the theory of local differential quasigroups and loops, and the theory of local algebras are discussed. Here, we give a definition of n-ary k-web and prove some properties.

Let Q be an arbitrary finite or infinite set. A mapping $f : Q^n \to Q$ is called an *n-ary operation* defined on Q. A set $\{f_1, \ldots, f_n\}$ of n-ary operations defined on Q is called orthogonal if for arbitrary elements $a_1, \ldots, a_n \in Q$ the system

$$\begin{cases} f_1(x_1, \ldots, x_n) = a_1, \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ f_n(x_1, \ldots, x_n) = a_n \end{cases}$$

has a unique solution.

Definition 1. A k-web W of the arity n consists of a set $\mathcal{P}$ of points and a subset $\mathcal{L}$ of the power set of $\mathcal{P}$ whose elements are called lines. In $\mathcal{L}$, there are k subsets called pencils ($k > n$) such that the following axioms hold:

(W1) Each line belongs to just one of the pencils of W.
(W2) Each point belongs to just one line from each pencil.
(W3) Any n lines from distinct pencils have exactly one point in common, whereas lines from the same pencil are disjoint.

Lemma 2. *There is a one-to-one correspondence between: 1) the points of a line and the lines of a pencil; 2) the lines of two arbitrary pencils; 3) the points of two arbitrary lines.*

Thus, the point set of a line and the line set of a pencil have the same cardinality m called *order* of the web. Since the lines of a pencil are disjoint, then the cardinal number of $\mathcal{P}$ is m^n. Let Q be a set of the order m, i.e., $|Q| = m$. Let us bijectively label all lines of each pencil by elements of the set Q and all points of the web by elements of the set Q^n. An expression $P(a_1, \ldots, a_n)$ means that the point P is labeled by $(a_1, \ldots, a_n)$ which is also called *coordinates* of P.

For each pencil $\mathcal{L}_i$, we define an n-ary operation f_i on the set Q: $f_i(a_1, \ldots, a_n) = a$ if the point with the coordinates $(a_1, \ldots, a_n)$ belongs to the line from $\mathcal{L}_i$ which is labeled by the element $a \in Q$. The obtained system $\Sigma := \{f_1, \ldots, f_k\}$ of operations are called *coordinate system of operations* or *coordinate operation system* (COS) of the web.

Theorem 3. *A set of k n-ary operations ($k > n$) is a coordinate operation system of an n-ary k-web if and only if it is orthogonal.*

References

[1] Belousov V. D. Configurations in algebraic webs. Kishinev, Stiintsa, 1979. 143 p. (in Russian).
[2] Akivis Maks A. ; Goldberg Vladislav V. Algebraic aspects of web geometry. Commentationes Mathematicae Universitatis Carolinae, Vol. 41 (2000), No. 2, 205–236.