Singularities of curves with two-parameter families of ideals

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We consider the study of ideals of commutative rings, in particular, the question about parametrization of classes of ideals is of current importance for the modern algebra.

Recall that a plane curve singularity over a field k is a k-algebra of the form

$$R = {}^{k[[x,y]]}/_{(f)}.$$

It is called one branch if R has no zero divisors.

Till now almost nothing was known about the curve singularities with at most m-parametric families of ideals if $m > 1$, in particular, how it relates with the Arnold's classification of singularities.

Sufficient and necessary conditions of possessing of one branch curve singularities at most 2-parameter families of ideals were researched.

Theorem 1. *If R is one branch singularity. Then the following conditions are equivalent:*

1) R has as maximum two parametric family of ideals.

2) If $\operatorname{char} k \neq 2$, then R dominates one of the following singularities:

$$E_{30},\ E_{32},\ W_{24}, W^{\#}_{2,*},\ N_{30},\ N_{20},\ N_{24},\ N_{28};$$

2a) If $\operatorname{char} k = 2$, then R dominates one of the following singularities:

$$E_{30},\ E_{32},\ W_{18}, W^{\#}_{1,*},\ N_{20},\ N_{24}.$$

Thus, it is proved sufficient and necessary conditions for a one-branch curve singularity S has at most two-parameter families of ideals.

References

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