Topological properties of Morse-Smale flows on a compact surface with boundary

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In this paper we consider the Morse flows [1] (Morse-Smale flows without closed orbits) on the compact surfaces with boundary. There was constructed a complete topological invariant of these flows – an equipped three-colored graph.

The graph T will be called *three-color graph*, if all its vertices have a degree not bigger 3, and edges are painted in three colors, so that edges of different colors converge at each vertex. Colors are denoted by the letters s, t, u.[2, 3] The vertices of three-colored graph correspond to the standard areas on the surface, that look like a curvilinear triangle or quadrilateral. There were found conditions in which a three-colored graph generates a flow.

Theorem 1. *For a connected tricolor graph having properties*

1) each edge of the graph is marked with one of the three letters: s, t, u, and each vertex is white or black;

2) two edges of the same type can not come out from each vertex;

3) for each black inner vertex there is a su -cycle of length 4 that contains it;

4) if two black vertices are connected by a $u-$ or $s-$ edge and one of them is bounded, then the other will be bound;

5) each white vertex is internal. And if it is connected to the black vertex $u-$ edge ($s-$ edge), then this black vertex will be the limit.

there exists a Morse flow on a connected surface with a boundary, the three-color graph of which is a given graph.[1]

The number of topologically non-equivalent flows with 2, 3, 4, and 5 standard areas was calculated. For each of them, the surface on which this flow is set is determined.

References

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