Right-angled polytopes, hyperbolic manifolds and torus actions

Taras Panov
(Lomonosov Moscow State University, Moscow, Russia)
E-mail: `tpanov@mech.math.msu.su`

A combinatorial 3-dimensional polytope P can be realised in Lobachevsky 3-space with right dihedral angles if and only if it is simple, flag and does not have 4-belts of facets. This criterion was proved in the works of Pogorelov and Andreev of the 1960s. We refer to combinatorial 3-polytopes admitting a right-angled realisation in Lobachevsky 3-space as Pogorelov polytopes. The Pogorelov class contains all fullerenes, i.e. simple 3-polytopes with only 5-gonal and 6-gonal facets.

There are two families of smooth manifolds associated with Pogorelov polytopes. The first family consists of 3-dimensional small covers of Pogorelov polytopes P, also known as hyperbolic 3-manifolds of Loebell type. These are aspherical 3-manifolds whose fundamental groups are certain finite abelian extensions of hyperbolic right-angled reflection groups in the facets of P. The second family consists of 6-dimensional quasitoric manifolds over Pogorelov polytopes. These are simply connected 6-manifolds with a 3-dimensional torus action and orbit space P. Our main result is that both families are cohomologically rigid, i. e. two manifolds M and M' from either family are diffeomorphic if and only if their cohomology rings are isomorphic. We also prove that a cohomology ring isomorphism implies an equivalence of characteristic pairs; in particular, the corresponding polytopes P and P' are combinatorially equivalent. This leads to a positive solution of a problem of Vesnin (1991) on hyperbolic Loebell manifolds, and implies their full classification.

Our results are intertwined with classical subjects of geometry and topology such as combinatorics of 3-polytopes, the Four Colour Theorem, aspherical manifolds, a diffeomorphism classification of 6-manifolds and invariance of Pontryagin classes. The proofs use techniques of toric topology.

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