Realization problems for Reeb graphs and epimorphisms onto free groups

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The Reeb graph $\mathcal{R}(f)$ of a Morse function f on a manifold is obtained by contracting connected components of its level sets. Sharko and Masumoto–Saeki showed that each graph with the so-called good orientation is the Reeb graph of a function on a closed surface. In this talk we focus on the problem of realization of a graph as the Reeb graph of a function on a given manifold M. In particular, it turns out that the maximal number of cycles among all Reeb graphs of functions on M is equal to the corank of $\pi_1(M)$, i.e. the maximal rank r for which there is an epimorphism $\pi_1(M) \to F_r$ onto the free group of rank r. It leads to the natural problem of representing any such epimorphism by the homomorphism induced on fundamental groups by the quotient map $M \to \mathcal{R}(f)$ for a Morse function f. We describe connections between Reeb graphs, epimorphisms onto free groups and systems of nonseparating 2-sided submanifolds. This allows us to study algebraic properties of epimorphisms onto free groups, such as their equivalence classes or ranks of maximal epimorphisms. For example, using theorems on topological conjugation of simple Morse functions on orientable (Kulinich, Sharko) or non-orientable surfaces (Lychak–Prishlyak), we may repeat some results on equivalence classes of epimorphisms onto free groups for surface groups provided by Grigorchuk–Kurchanov–Zieschang.

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