On orbit braids

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Let M be a connected topological manifold of dimension at least 2 with an effective action of a finite group G. Associating with the orbit configuration space $F_G(M, n), n \geq 2$ of the G-manifold M, we try to upbuild the theoretical framework of orbit braids in $M \times I$ where the action of G on I is trivial, which contains the following contents: We introduce the orbit braid group $\mathcal{B}_n^{orb}(M, G)$, and show that it is isomorphic to a group with an additional endowed operation (called the extended fundamental group), formed by the homotopy classes of some paths (not necessarily closed paths) in $F_G(M, n)$, which is an essential extension for fundamental groups. The orbit braid group $\mathcal{B}_n^{orb}(M, G)$ is large enough to contain the fundamental group of $F_G(M, n)$ and other various braid groups as its subgroups. Around the central position of $\mathcal{B}_n^{orb}(M, G)$, we obtain five short exact sequences weaved in a commutative diagram. We also analyze the essential relations among various braid groups associated to those configuration spaces $F_G(M, n), F(M, n)$, and $F(M/G, n)$. We finally consider how to give the presentations of orbit braid groups in terms of orbit braids as generators. We carry out our work by choosing $M = \mathbb{C} \approx \mathbb{R}^2$ with typical actions of $\mathbb{Z}_p$ and $(\mathbb{Z}_2)^2$. We obtain the presentations of the corresponding orbit braid groups, from which we see that the generalized braid group $Br(B_n)$ (introduced by Brieskorn) actually agrees with the orbit braid group $\mathcal{B}_n^{orb}(\mathbb{C} \setminus \{0\}, \mathbb{Z}_2)$ and $Br(D_n)$ is a subgroup of the orbit braid group $\mathcal{B}_n^{orb}(\mathbb{C}, \mathbb{Z}_2)$. This is a joint work with Hao Li and Fengling Li